A behavioral model for mutual fund dynamics: structural estimation approach

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Abstract

Based on Berk and Green (2004), I develop a model that explains the following well-known stylized facts on mutual funds in a unified framework: (i) a negative aggregate return, (ii) a short-term return persistence, and (iii) a convex return-flow relationship. In the model, agents learn about managers' time-varying abilities from fund returns and non-return information signals. Under decreasing returns to scale, investors equilibrate expected fund returns through fund flows, but their expectations are biased due to overconfidence about precision of non-return signals and overextrapolation of past return trends. I employ a Simulated Method of Moments (SMM) to estimate the model parameters. The model matches most of the 15 moments, and is not rejected at the 10% level. I run a horse race between rational equilibrating forces and behavioral inefficiencies by allowing parameters for biases determined by data. As a result, both information processing biases appear to be important to generate a negative aggregate return and short-term return persistence, and to improve a model fit.

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1. Introduction

Mutual funds constitute the largest vehicle for individual investors to participate in the US stock market. While passive mutual funds and ETF markets have grown, the total assets that active mutual funds manage still dominate the size of passively managed mutual funds. However, historically active mutual funds have performed worse than their passive benchmarks after fees, causing researchers to question whether investing in active mutual funds is worthwhile. It is puzzling that the active funds still receive remarkable capital despite their continued underperformance at the aggregate level. To justify the value of active mutual funds, scholars argue that a subset of mutual fund managers may have the ability to beat the market persistently. However, they have not yet found convincing results of persistent superior performance.

Berk and Green (2004) claim that the absence of persistence in positive performance is a natural finding. Investors choose the size of fund flows to maximize their expected return on investments, and the expected returns of funds become equal in equilibrium under decreasing returns to scale. There is no performance continuation even for highly skilled managers. But in this model, we cannot explain why we find the aggregate underperformance of active funds, as Fama and French (2010) argue.[1]

Researchers have primarily concentrated on the debate of fund returns, rather than other dynamics of mutual funds. However, other fund dynamics also have to be the test criteria of the mutual fund model. The well-known stylized facts of mutual fund dynamics, that is, convexity in return-flow relationship, negative aggregate return, short-term return persistence[2] and fund survivorship, are related to each other and have to be explained in a single framework.[3]

In this paper, I develop a model that explains the following well- known stylized facts on mutual funds in a unified framework: (i) a negative aggregate return, (ii) a short-term return persistence, and (iii) a convex return-flow relationship. The model allows for a horse race between rational equilibrating forces and behavioral inefficiencies.[4] On the one hand, the model succeeds the equilibrating force of Berk and Green (2004) model - money should

[1]Fama and French (2010) argue that active mutual fund managers do not have any ability to beat the market. The aggregate holding of active mutual funds is almost the same as the market portfolio, and the actual fund return distribution is not significantly different from the distribution driven by luck. Berk and Binsbergen (2012) find the empirical evidence that the management fees are as large as the risk-adjusted profits for the international mutual funds, but still cannot rationalize the findings for US equity funds.

[2]A risk-adjusted fund return has a predictability about 10% by its lagged return as shown in the table 3.

[3]Fama and French (2010) have only focused on the negative aggregate return while ignoring other dynamics. Berk and Green (2004) explain the convex return-flow relationship and the survivorship probabilities, but fail to explain negative aggregate return and return predictability.

[4]The data enable us to distinguish influences of these two forces.

flows into the funds which are predicted to have more abilities. On the other hand, the model contains behavioral inefficiencies - investors have biased expectations and misallocate capital due to overconfidence about precision of non-return signals and overextrapolation of past return trends. These two forces compete and determine mutual fund dynamics including above stylized facts.

In the model, agents cannot observe manager's ability directly, thus they have to infer time-varying managerial abilities from mutual fund returns and non-return information signals following Bayes' rule. The non-return information includes both hard and soft information about managerial abilities which are complementary to fund returns such as news, advertisements, brochures, their strategies, holdings, and characteristics. The expected fund returns are formed by updated believes about managerial abilities and expected agents' actions.

The model consists of two types of agents: fund managers and investors. Both agents are risk-neutral[5] and maximize their utilities. Investors maximize their expected investment returns each period by reallocating money among the funds. A fund manager chooses the optimal active share of the fund in order to maximize the total management fee. A fund incurs a convex trading cost to manage its assets actively, but no cost to invest in passive benchmark. A fund shows decreasing returns to scale accordingly. In equilibrium, investors are indifferent to invest among active funds and passive benchmark. Moral hazard of fund managers and other frictions are not considered in the paper.

I assume that managerial ability changes over time contrasting to Berk and Green (2004)'s non time-varying managerial ability assumption. A fund manager can improve her investing skill through her experience. Investing skill can also become outdated due to changes in the trading environment and a trading strategy that has worked previously may become unproductive. For this reason, I assume that the managerial ability process is mean-reverting with a persistent ability shock.

Another contrasting point is that agents suffer from behavioral biases in processing information - overconfidence and overextrapolation. Based on extensive psychological evidence and ample empirical evidence[6], agents are assumed to be overconfident and overextrapolative. Because inferring managerial abilities is a major concern of mutual fund investors,

[5]Investors are risk-averse only regarding the fundamental risks in stock returns, but not for the risk in managerial abilities. Investors are well diversified on idiosyncratic managerial ability shocks. After adjusting a mutual fund return to risk factors, the model can be simplified as risk-neutral.

[6]Regarding overconfidence, see e.g. Alpert and Raiffa (1982), Staël Von Holstein (1972), Daniel et al. (1998), Russo and Schoemaker (1992), Ben-David et al. (2013), Odean (1998), Daniel et al. (1998), Scheinkman and Xiong (2003), and Alti and Tetlock (2012). Regarding overextrapolation, see e.g. Tversky and Kahneman (1974), Haruvy et al. (2007), Greenwood and Nagel (2009), Choi et al. (2010), Benartzi (2001), La Porta et al. (1997), Barberis et al. (1998), Hirshleifer and Yu (2012), and Alti and Tetlock (2012).

information processing biases are most relevant among behavioral biases and play significant roles in mutual fund dynamics. I model overconfident investors as they are more confident about their non-return information signals than the actual precision of the signals following Odean (1998), Scheinkman and Xiong (2003), and Alti and Tetlock (2012). I assume that overextrapolative agents perceive the persistence of managerial ability change is greater than the true persistence as in Barberis et al. (1998), Hirshleifer and Yu (2012), and Alti and Tetlock (2012).

Both biases help to match the observed level of negative aggregate return and return predictability. Overconfidence generates negative aggregate returns and return predictability. Overconfident agents underutilize information from the historical fund return. When there is a positive or negative fund return, an investor relies too much on the current non-return information and does not properly reevaluate past fund returns. As a result, funds with negative returns remain negative on average in the next period. This mistake is symmetric for positive returns and negative returns but the funds with negative returns are larger than the funds with the positive return. The value weighted fund return becomes negative due to this asymmetric size effect[7] and the fund returns persist at the individual fund level.

Overextrapolative agents allocate excessive money into these funds that they perceive as being better than the average fund because they think that the superior ability will persist. Overextrapolation causes the estimation error to be biased, i.e. the perceived managerial ability is higher than the true ability for the above-mean manager and vice versa. The resulting value-weighted fund return is negative due to estimation bias combined with the asymmetric size effect.[8] At the individual fund level, investors continuously allocate excessive money into the same funds because of this persistent belief, and thus overextrapolation produces a positive return predictability.

Both biases appear to have same directional effects for aggregate returns and return predictability. But overconfidence cannot generate enough negative aggregate returns and overextrapolation cannot generate return predictability while matching other moments in the data. In the overconfidence-only model, initial misallocation is too small to have large enough asymmetric size effects, causing small negative aggregate returns.[9] In the overextrapolation-only model, opposite signs of next period returns diminish positive persistence. Aggressive allocation in the current period leads to the opposite sign of next period returns and new updates of returns cancel out a part of persistent believes. As a result, overextrapolation by itself cannot produce enough return predictability. The interaction effect of these two

[7]But the magnitude of negative return is tiny at the plausible overconfidence level.

[8]The funds with the negative return are larger than the funds with the positive return.

[9]The magnitude of asymmetric size determines the magnitude of aggregate underperformance.

biases is not just additive. Overextrapolation makes agents misallocate capital initially and overconfidence deters a correction mechanism for the misallocation driven by overextrapolation. The interaction effect allows the model to match the empirical data with a reasonable parameter values.

The model also contains the convex return-flow relationship as in Berk and Green (2004). Under a convex trading cost, a fund manager reduces the active share as perceived ability rises in order to maximize the fund size, which is equivalent to maximizing his compensation. As the active share decreases, the fund size capacity to equilibrate the expected return increases convexly in observed fund returns. Therefore, the magnitude of inflows to funds with positive returns is larger than the magnitude of outflows from funds with negative returns.

I structurally estimate the model using the Simulated Method of Moments (SMM). Mutual fund dynamics are difficult to test in a reduced form model because fund return and size have an endogenous relationship, information processing is hidden, and relationships are nonlinear. In this sense, a structural estimation is an appropriate empirical method. I utilize mainly three stylized facts: negative aggregated return, return predictability, and convexity in return-flow relationships. In addition to these stylized facts, I also use return, flow and size cross-sectional distributions, and fund survivorship probability and return. There are 11 parameters to be estimated, and the number of moments is 15. The model is over-identified and allows for a test statistic. In each simulation, I generate 20,000 funds over 45 years, which is the length of the sample observation period. I repeat simulation 10,000 times to complete the estimation.

A total of 15 moments are selected for economic reasons, but they well identify the parameters. 8 moments are from three stylized facts and other 7 moments consist of cross-sectional distribution and fund survivorship. The model successfully matches negative aggregate returns, return predictability, size, flow and survivorship moments. The model has a p-value of 0.1994 and is not rejected at the 10% level. The model with only overconfidence or no bias are rejected at the 1% level and the model with only overextrapolation is rejected at the 5% level.

All the estimates of the model parameters are in reasonable range. I find that the median of the manager's ability is 22.03% on the first dollar invested. The results should be interpreted cautiously because the estimation does not yet consider the adverse size effect. The median marginal ability (22.03%) corresponds to the fund size of 215 million dollars that this manager afford to run without any excess returns after fees. The marginal ability of the manager who manages one billion dollar is 45.28% and that of 100 million dollar fund is 15.38%. The mutual fund investors exhibit overconfidence to their non-return information signals. The estimate of perceived precision of non-return information signal

(0.7554) is greater than the estimate of true precision (0.3232). However, the degree of overconfidence is milder than the degree of overconfidence in surveys which are designed to study overconfidence. The agents also display a modest level of overextrapolation.[10]

As an out-of-sample test, I compare the return persistent patterns between simulated data and actual data up to five years. I also compare the returns of flow quintile groups between actual data and simulated data. The simulated return patterns resemble the actual patterns for both return persistence and flow-return relation. For the return persistence, return spreads decrease greatly one year after the ranked year and returns become indistinguishable afterwords across return groups. For the flow-return relation, flows do not predict returns as in the data. The result can be interpreted as a rational variation dominates a behavioral variation, because flows are mixed response of rational force and behavioral force in the model.

From the simulated data, I find that the current fund size is relatively too large in comparison to the managers' capability to manage it. The size of misallocated funds is 8.58% of total fund size. The optimal aggregate fund size from the rational equilibrium is about 4.63% less than the current aggregate fund size. The overallocation to active funds causes over-concentration in US equities among asset classes. This overinvestment in the US stock market can be calculated as overallocation to active mutual funds (4.63%) times the fraction of stock market capital owned by active mutual funds (16.9%), and it is 0.78% of the US equity market capitalization.[11] In other words, 0.78% of equity capital is overvalued due to behavioral biases of mutual fund investors.

The primary contribution of this paper is developing a model to explain mutual fund dynamics. The model successfully connects different aspects of mutual fund dynamics and exhibits high explanatory power for the cross-section of mutual fund sizes and returns. Specifically, the model produces (i) moderate negative aggregate fund returns and (ii) individual fund return predictability, while matching cross-sectional flow, size and return distributions. By structurally estimating the parameters of the model, I identify the magnitude of these information processing biases from observed mutual fund dynamics. My estimation further confirms decreasing returns to scale and the existence of equilibrating forces through flows. The estimation results suggest that mutual fund managers have skills, but the misallocation of funds results in aggregate negative performance.

The paper is organized as follows. Section 2 presents a literature review. Section 3 develops the model with Bayes' learning. Section 4 describes data and moment selection. Section 5 presents estimation strategy and estimation results. Section 6 conducts the out-

[10]The level of overextrapolation is 1.5% and is small relative to the one-year ability persistence (93.2%).

[11]The fraction of stock market cap owned by active mutual funds is 16.9% at the end of year 2012.

of-sample tests and reports their results. Section 7 presents the implications of the paper. Section 8 concludes.

2. Literature review

My paper is related to several strands of mutual fund research - negative risk-adjusted return of mutual funds, return persistence, convexity of return-flow relationship, decreasing returns to scale, and behavioral bias of mutual fund investors.

The debate over the performance of mutual fund has lasted over almost a half century since Sharpe (1966). French (2008) argues that mutual fund managers do not generate positive profit after the fees they gather. Even performance before fee is not distinguishable compared to passively managed index funds. Fama and French (2010) claim that empirical evidence that at least a subset of managers have ability is just type 1 error and coincides with the case that no fund manager has ability under their distributional assumptions. Berk and Binsbergen (2012) find an opposite evidence to Fama and French (2010) when they extend the mutual fund sample to international equity funds. They show that management fees which managers earn are almost same as excess profits from trading of funds. In the data, I find the underperformance of US equity funds similar to French (2008).

Regarding return persistence[12], Carhart (1997) finds that funds do not display persistence in their return. He tracks the excess returns on the decile portfolio of previous year performance rankings up to next five years. Performance difference of top decile and bottom decile portfolios becomes insignificant after two years. Bollen and Busse (2004) tests persistence of mutual fund performance using quarterly measured performance. The post-ranking abnormal return disappears for longer evaluation periods. They also find that risk-adjusted return sorts have return persistence up to one year horizon. Berk and Green (2004) claim that the performance of mutual funds does not persist because the new flows in and out of funds change expected returns of funds and expected returns are same at the equilibrium. Their model does not allow even a short-term persistence. In the data, I find short-term persistence up to one year as in Bollen and Busse (2004).[13] Regarding a convex return-flow relationship, Chevalier and Ellison (1997) and Sirri and Tufano (1998) show a convex relation between mutual fund flows and past returns.[14] I also find the convex relation between

[12]There are also several papers related to mutual fund return predictability. Cremers and Petajisto (2009), Kacperczyk et al. (2005) and Kacperczyk et al. (2006) find that proxies of activeness predict higher performance.

[13]I find that raw return sorts have return persistence up to one year horizon as well as risk-adjusted return sorts.

[14]Sirri and Tufano (1998) indicate search costs as the cause of convexity.

fractional flows and past risk-adjusted returns.

After Berk and Green (2004) assume diseconomies of scale of mutual funds, Chen et al. (2004) empirically test the existence of diseconomies of scale. They validate the assumption of Berk and Green (2004) and argue that liquidity is the main reason for adverse scale effect.[15] Pollet and Wilson (2008) also indicate strategy change is another reason of diminishing returns to scale. Pastor and Stambaugh (2012) argue that funds show decreasing returns to scale in industry sizes, and rationalize a negative performance puzzle in an economy with rational investors.[16] Pástor et al. (2014) find that industry adverse size effects are larger than individual fund size effects.[17] Insignificant correlation between aggregate fund returns and aggregate industry sizes may suggest no industry adverse size effects. But, after accounting for the fact that better skilled funds enter to industry over times, they find an industry level diseconomy of scale. My paper is not necessarily contradicting Pástor et al. (2014)'s result because my model implies that it is difficult to find individual adverse size effect in panel regression due to equilibrating forces. Industry sizes would capture the time-varying degree of biases in my model. In that case, fund returns show decreasing returns to scale in industry sizes.

There has been much research on overconfidence.[18] I model overconfident agents following Odean (1998), Scheinkman and Xiong (2003), and Alti and Tetlock (2012). The papers distinguish public signals and private signals and agents perceive larger precision on a private

[15]However, fund sizes cannot predict fund returns in Berk and Green (2004)'s model.

[16]They claim that 45 years are not enough time for rational investors to learn mean managerial abilities and parameters of decreasing returns to scale. The argument relies on the small variation in relative industry size over 45 years of mutual fund observation. They define an industry size as total active mutual fund size over total mutual fund size. But if they use alternative industry size variable as active mutual fund size over the total stock market capitalization as in Pástor et al. (2014), it would be much more difficult to rationalize underperformance because of larger variation in industry size variable.

[17]Pástor et al. (2014) argues that Chen et al. (2004) can have endogeneity issue. The main regression of Chen et al. (2004) is:

$$R_{i,t}=a_i+\beta S_{i,t-1}+\varepsilon_{i,t}$$

where $R_{i,t}$ is a fund return and $S_{i,t-1}$is a fund size.

Skill is unobservable and related to both $R_{i,t}$ and $S_{i,t-1}$. But in general, we think that higher skill is related to larger fund size since manager has incentive to run larger mutual funds due to a fee structure. If the correlation between skills and fund sizes is positive, omitting skills from the regression imparts a positive bias in the estimate of β. The assumption of positive correlation is not odd and the endogeneity issue do not undermine their findings.

[18]When limited to business areas, Alpert and Raiffa (1982), Staël Von Holstein (1972), Russo and Schoemaker (1992), and Ben-David et al. (2013) conduct surveys of Harvard Business School students, money managers, investment bankers, and executives respectively. Odean (1998) develop a model that overconfidence increases trading volume and market depth and decreases the expected profit of overconfident traders. Barber and Odean (2001) empirically confirm that overconfidence drives excessive trading of stock investors. Daniel et al. (1998) establish a model that overconfidence results in negative long-lag autocorrelations, excess volatility, and public-event-based return predictability in the stock market.

or a soft signal than the actual precision. In my paper, agents are overconfident on non-return information signals. Hirshleifer and Yu (2012)[19] document well about overextrapolation.[20] I model overextrapolative agents following Barberis et al. (1998), Hirshleifer and Yu (2012) and Alti and Tetlock (2012). My paper is also related to research about behavioral biases among investors. Bailey et al. (2011) investigates mutual fund investors' behavioral biases using individual level data. In my paper, I make parametric assumptions for behavioral biases and use the aggregate quantities to infer the degree of biases in contrast with Bailey et al. (2011).

Few papers estimate mutual fund dynamics structurally. Linnainmaa (2013) estimates the mutual fund dynamics using simulated method of moments.He argues that underperformance of mutual funds are attributable to the dead funds while they have higher abilities than realized returns. He utilizes alpha estimates for disappearing funds and surviving funds, and fraction of funds disappearing for different ages of funds as moments. Berk and Green (2004) calibrate their model and generate return-flow relationship and survivorship graphs similar to actual graphs. Compared to these papers, my paper utilizes more stylized facts of mutual funds, and produces a more complete picture of mutual fund dynamics.

3. Model

The model extends Berk and Green (2004) by adding two additional components: time-vayring managerial abilities and information processing biases.[21] There are two types of agents in the economy: fund managers and investors. Fund managers are risk-neutral and maximize their utilities. Fund managers are compensated based on management fees which have only a variable part proportional to fund sizes. Fund managers' objective function is equivalent to maximizing fund sizes due to risk-neturality of fund managers and a management fee structure. Fund managers choose the active share of assets under managements at each period. They can allocate part of their asset in passive benchmark without cost.

[19]Hirshleifer and Yu (2012) argue that a standard model with recursive utility combining with overextrapolation can reconciles the stylized facts about financial markets and real business sectors.

[20]Tversky and Kahneman (1974) find that individuals follows individuals following the representativeness heuristic, and thus overweight small number of observations when they update their believes. Several studies empirically show that retail and professional investors overextrapolate past performance. (e.g., Haruvy et al. (2007), Greenwood and Nagel (2009)) Choi et al. (2010) shows that investors of S&P 500 index funds overextrapolate annualized returns before fees and choose high-fee funds suboptimally in a lab experiment. Benartzi (2001) empirically finds overextrapolation of 401(k) accounts in their investments to stocks. La Porta et al. (1997) find the evidence of overextrapolation from stock price reactions around earnings announcements for value and growth stocks.

[21]I adopt Berk and Green (2004)'s notations.

Investors are also risk-neutral.[22] Investors decide investment allocation among funds to maximize expected returns of investments. The decision variables of the model are a fund allocation (S_t) and active fund size (A_t), where S_t is a choice variable of investors and A_t is a choice variable of fund managers.

A manager's ability follows a mean-reverting process (AR(1)) with homogenous mean across funds:

$$m_{t+1} = m_t + \rho\left(\bar{m} - m_t\right) + \sigma_m \varepsilon_{t+1}^m \tag{1}$$

where m_t is a managerial ability at time t, σ_m is a magnitude of manager's ability shock and $\bar{m}$ is a mean managerial ability. Managerial ability shocks are not transitory, but rather persistent. Persistence of shocks is governed by a mean reversion parameter ρ. Manager's ability corresponds to the return of first dollar actively invested.

Agents do not know initial managers' abilities and also do not observe managerial ability changes. They only know an initial distribution of managerial abilities and a distribution of managerial shocks. They estimate managers' abilities by observing a history of fund returns and by receiving non-return information regarding managers' ability changes. Non-return information contains both hard and soft information about managerial abilities which complement information from fund returns, and is homogenous to all agents. Non-return information includes news articles, advertisements, brochures, strategies, holdings, and characteristics of mutual funds. Loads, tax, moral hazard and other frictions are not included in the model. There are enough unconstrained investors and competitive provision of capital by these investors.

Active mutual funds incur costs to manage their assets actively. Management costs are mainly associated with liquidities and price impacts. Large trading volume adversely affect stock price and thus a profit per dollar of mutual funds decreases with sizes of trading. Trading strategies of larger funds are more easily exposed to others, and therefore profits of strategies decrease in fund sizes. For these reasons, I assume that trading costs are increasing and convex in actively managed fund sizes. A trading cost function ($C\left(A\right)$) is only a function of actively manged fund sizes and is assumed to be homogenous across funds, even though styles of funds matter for diseconomies of scale.[23] A management fee (f) is exogenously

[22]Investors are risk-averse only regarding the fundamental risks in stock returns, but not for risks in managerial abilities. In other words, managerial abilities do not have systematic risks. Investors are well diversified on idiosyncratic managerial ability shocks. Investors can make the statistical arbitrage portfolio, and assets with higher risk-adjusted return are always preferred. After adjusting mutual fund returns to risk factors, the model is simplified as risk-neutral. Investors can be assumed to be risk-averse to managerial ability shocks. Investors would require positive risk-premium, and then it worsen the negative aggregate return puzzle. To be more conservative, I assume risk-neutrality.

[23]Small size or small growth category funds face more diseconomies of scale because liquidities of those stocks are low.

determined and fixed per dollar unit of funds' total net assets.

Fund returns are total payouts to investors divided by ex-ante total net assets. Total payouts to investors are net of profits from active management, trading costs and management fees:

$$\mathrm{TP}_{t+1} = S_t a(S_t) R_{t+1} - C(S_t a(S_t)) - S_t f \tag{2}$$

where TP_{t+1} is an excess total payout to investors over benchmarks, R_{t+1} is an active return in excess of passive benchmark returns, S_t is a fund size, $a(S_t)$ is an active share which is a fraction of actively managed funds and $C(A_t)$ is costs of management which is a function of active fund size. A fund return (r_{t+1}) is

$$\begin{aligned} r_{t+1} &= \frac{\mathrm{TP}_{t+1}}{S_t} \\ &= a(S_t) R_{t+1} - \frac{C(S_t a(S_t))}{S_t} - f \\ &= a(S_t) R_{t+1} - c(S_t) \end{aligned} \tag{3}$$

where $c(S_t) = \frac{C(S_t)}{S_t} + f$ is a unit cost of funds including a management fee. R_t is an excess return of actively managed assets. A fund return is a combined return of an actively managed portion and a passively managed portion netting trading costs and management fees. An active share is decided optimally by fund managers, and can be expressed as a function of fund sizes (S_t). An active return is defined as a sum of managerial abilities and active return errors, $R_{t+1} = m_{t+1} + \sigma_e \varepsilon^e_{t+1}$, $\varepsilon^e_{t+1} \sim \mathrm{N}(0,1)$, where m_{t+1} is a managerial ability, and σ_e is a magnitude of active return error.

In equilibrium, investors allocate money up to the point where the expected excess fund returns become zero, $E_t(r_{t+1}) = 0$, because investors are assumed to have a homogenous expectation on managerial abilities and have sufficient capitals to invest competitively. Managerial ability distribution is exogenously determined.[24]

At each period, fund managers maximize total amount of management fees by choosing actively managed fund size (A_t). In equilibrium, total fees are equal to expected total profits from fund operations. The equilibrium condition can be written as:

$$S_t f = A_t \hat{m}_{t+1} - C(A_t). \tag{4}$$

, where $\hat{m}_{t+1}$[25] is the estimated managerial ability of time t+1 at time t. Managers'

[24] The model is a partial equilibrium in the sense that agents' actions do not affect benchmark returns.

[25] $\hat{m}_{t+1}$ is the outcome of investors' learning, and is a function of $\hat{m}_t$, return information and non-return information. It is defined in equation 8.

optimization problem can be expressed as $\max_{A_t} A_t\hat{m}_{t+1} - C(A_t)$. The first order condition of the problem is

$$\hat{m}_{t+1} = C'(A_t). \tag{5}$$

An active fund size (A_t) is a function of a perceived manager's ability, $A_t = C'^{-1}(\hat{m}_{t+1})$. The equilibrium condition can be written as $S_t f = A_t\hat{m}_{t+1} - C(A_t) = C'^{-1}(\hat{m}_{t+1})\hat{m}_{t+1} - C(C'^{-1}(\hat{m}_{t+1}))$ using the FOC. We can express S_t as $S_t = (C'^{-1}(\hat{m}_{t+1})\hat{m}_{t+1} - C(C'^{-1}(\hat{m}_{t+1})))/f$. An expected managerial ability ($\hat{m}_{t+1}$) is mapped by a fund allocation (S_t), and therefore an active share, $a(S_t) = A_t/S_t$ is also expressed as a function of a fund size (S_t). In the later subsection, the equilibrium equations will be parametrized using a specific form of cost function. A percentage fund flow is defined as new fund allocations over ex-ante fund sizes[26]:

$$\text{flow}_{t+1} = \frac{S_{t+1} - (1 + r_{t+1}) S_t}{S_t}. \tag{6}$$

The mutual fund model in the paper is a discrete time model. Information about mutual funds is less frequent than information about stocks. For example, awards or rankings for funds is mostly available on the annual basis and news about the funds do not appear more frequently than news about companies. Advertisements and brochures of mutual funds are updated on the annual basis. As an investor, front-end and back-end load deter frequent update of managerial abilities and reallocation of their investments. The mutual fund empirics in the literature are also based on annual data for these reasons. For example, a convex relationship between fund returns and flows is an one-year response of investors to a previous one-year return. Funds display a return persistence at one-year horizon. Simulated moments using a discrete time filtering process are naturally matched to empirical moments. A discrete time model is more suitable for this research than a continuous time model is.

Managerial abilities are unobservable to agents, and therefore agents need to infer managerial abilities from observable information signals. I assume that agents can observe active returns of funds and receive non-return information signals at each period. An expected managerial ability ($\hat{m}_{t+1}$) is acquired by Bayesian learning process. The expected fund return can be expressed as:

$$\begin{aligned} E_t(r_{t+1}) &= a(S_t) E_t(R_{t+1}) - c(S_t) \\ &= a(S_t)\hat{m}_{t+1} - c(S_t). \end{aligned} \tag{7}$$

Learning about managerial abilities is a nonstationary discrete filtering problem. Agents observe two different signals - a fund return signal and a non-return signal- and update

[26] Berk and Green (2004) define fund flows as $\text{flow}_{t+1} = \frac{S_{t+1} - S_t}{S_t}$

their believes. A fund return can be decomposed as three components: an active return, a trading cost associated with a fund size, and an additional fund return error, that is, $r_{t+1} = a(S_t) R_{t+1} - c(S_t) + \sigma_r \varepsilon_{t+1}^r$. Additional fund return errors are errors in active fund returns and trading costs which investors can comprehend. A non-return information signal is a noisy signal for managerial ability change: $s_t = \eta \varepsilon_t^m + \sqrt{1-\eta^2}\varepsilon_t^s$, where η denotes a signal informativeness and ε_t^s is a non-return information signal noise. A signal precision θ is defined as

$$\theta = \frac{\eta}{\eta + \sqrt{1-\eta^2}}.$$

Error terms (ε_t^e, ε_t^m, ε_t^s, ε_t^r) are independent identically distributed following a normal distribution with mean zero and variance one. The estimated managerial ability $\hat{m}_{t+1}$ is described as the following process[27]:

$$\hat{m}_{t+1} = (1-\rho)\hat{m}_t + \rho\bar{m} + \zeta_t \left(R_{t+1} - \left((1-\rho)\hat{m}_t + \rho\bar{m}\right)\right) + \psi_t s_{t+1} \tag{8}$$

$$\zeta_t = \frac{(1-\rho)^2 P_t + \sigma_m^2 - \eta^2\sigma_m^2}{\left((1-\rho)^2 P_t + \sigma_m^2 + \sigma_e^2\right) - \eta^2\sigma_m^2}$$

$$\psi_t = \frac{\eta\sigma_m\sigma_e^2}{\left((1-\rho)^2 P_t + \sigma_m^2 + \sigma_e^2\right) - \eta^2\sigma_m^2}$$

Estimation of managerial abilities follows a non-stationary process. In other words, variance of difference between actual and estimated managerial ability, $\text{Var}(m_t - \hat{m}_t)$, changes over time. The error variance process is

$$P_{t+1} = (1-\rho)^2 P_t + \sigma_m^2 - \left(\zeta_t \left((1-\rho)^2 P_t + \sigma_m^2\right) + \psi_t\eta\sigma_m\right) \tag{9}$$

Depending on initial magnitudes of estimation errors, estimation errors either decrease or increase, and eventually converge to a stationary error variance. I assume that manager's ability m_0 is drawn from $\text{N}\left(\bar{m}, \frac{\sigma_m^2}{\rho}\right)$, which is a stationary distribution of abilities from an AR(1) process of managerial abilities. A variance of initial estimation errors is σ_{m0}^2. An initial estimated manager's ability, $\hat{m}_0$, follows $\text{N}(m_0, \sigma_{m0}^2)$. The stationary error variance is $\gamma = \lim_{t\to\infty} P_t$.[28] If $\sigma_{m0}^2 > \gamma$, then error variances decrease over times and vice versa.

[27] Derivation is in the appendix.

[28] The steady state error variance is from the equation:

$$\gamma = (1-\rho)^2\gamma + \sigma_m^2 - \left(\zeta\left((1-\rho)^2\gamma + \sigma_m^2\right) + \psi\eta\sigma_m\right)$$

I set up a fund exit rule as perceived managerial ability hits an exit threshold as in Berk and Green (2004). The threshold abstracts an exit problem including entry costs and management fixed costs in a reduced form.

The time-line is as follows: At time t, a fund has a total net asset S_{t-1}. A fund return, r_t, is realized and agents receive a non-return information signal, s_t. Managers and investors update their believes about managerial abilities using observed fund returns (r_t) and non-return information signal (s_t). Agents can observe all historical returns of the fund up to time t. Investors decide their fund allocation (S_t) and managers adjust their active shares ($a(S_t)$) contingent on their updated believes about managerial abilities.

Behavioral biases

All agents are subject to two behavioral biases - overconfidence and overextrapolation - when they process information. I adopt the specification of biases of Alti and Tetlock (2012). I assume that agents are overconfident in a sense that they believe their non-return information is more accurate than its actual accuracy. The perceived non-return signal informativeness (η_B) is greater than the actual non-return signal informativeness (η). Overextrapolative agents think that managerial ability changes are more persistent than they actually are. Formally, they believe that a perceived mean reversion parameter (ρ_B) is smaller than an

where

$$\zeta = \frac{(1-\rho)^2\gamma + \sigma_m^2 - \eta^2\sigma_m^2}{\left((1-\rho)^2\gamma + \sigma_m^2 + \sigma_e^2\right) - \eta^2\sigma_m^2}$$

$$\psi = \frac{\eta\sigma_m\sigma_e^2}{\left((1-\rho)^2\gamma + \sigma_m^2 + \sigma_e^2\right) - \eta^2\sigma_m^2}$$

This will give the second order equation for γ

$$\begin{aligned}\left(\left((1-\rho)^2\gamma + \sigma_m^2 + \sigma_e^2\right) - \eta^2\sigma_m^2\right)\left(\left(1-(1-\rho)^2\right)\gamma - \sigma_m^2\right)& \\ +\left(\left(\left((1-\rho)^2\gamma + \sigma_m^2\right) - \eta^2\sigma_m^2\right)\left((1-\rho)^2\gamma + \sigma_m^2\right) + \eta\sigma_m\sigma_e^2\eta\sigma_m\right) &= 0\end{aligned}$$

$$(1-\rho)^2\gamma^2 + \left(\left(1-(1-\rho)^2\right)\sigma_e^2 + \left(1-\eta^2\right)\sigma_m^2\right)\gamma - \left(1-\eta^2\right)\sigma_m^2\sigma_e^2 = 0$$

γ can be expressed as:

$$\gamma = \frac{-\left(1-(1-\rho_b)^2\right)\sigma_e^2 + \left(1-\eta_b^2\right)\sigma_m^2 + \sqrt{\left(\left(1-(1-\rho_b)^2\right)\sigma_e^2 + \left(1-\eta_b^2\right)\sigma_m^2\right)^2 + 4(1-\rho_b)^2\left(1-\eta_b^2\right)\sigma_m^2\sigma_e^2}}{2(1-\rho_b)^2}.$$

actual mean reversion parameter (ρ). With behavioral biases, the learning process becomes

$$\hat{m}_{t+1}^{B} = (1-\rho_B)\,\hat{m}_t^B + \rho_B \bar{m} + \zeta_t \left(R_{t+1} - \left((1-\rho_B)\,\hat{m}_t^B + \rho_B \bar{m}\right)\right) + \psi_t s_{t+1} \tag{10}$$

$$\zeta_t = \frac{(1-\rho_B)^2 P_t^B + \sigma_m^2 - \eta_B^2 \sigma_m^2}{\left((1-\rho_B)^2 P_t^B + \sigma_m^2 + \sigma_e^2\right) - \eta_B^2 \sigma_m^2}$$

$$\psi_t = \frac{\eta_B \sigma_m \sigma_e^2}{\left((1-\rho_B)^2 P_t^B + \sigma_m^2 + \sigma_e^2\right) - \eta_B^2 \sigma_m^2}$$

$$P_{t+1}^B = (1-\rho_B)^2 P_t^B + \sigma_m^2 - \left(\zeta_t \left((1-\rho_B)^2 P_t^B + \sigma_m^2\right) + \psi_t \eta_B \sigma_m\right). \tag{11}$$

Parametrization

I assume that a cost function is a power function of fund sizes: $C(A) = bA^x$, where $x > 1$. An active fund size is expressed as a function of perceived managerial ability, $A_t = \left(\frac{\hat{m}_{t+1}^B}{bx}\right)^{\frac{1}{x-1}}$ which is from the FOC of manager's optimization.[29] The fund size is also summarized as a function of estimated managerial abilities, $S_t = \frac{(x-1)\hat{m}_{t+1}^{B\ \frac{x}{x-1}}}{b^{\frac{1}{x-1}} x^{\frac{x}{x-1}} f}$, which is driven from the equilibrium condition.[30] The active share is simplified as: $a(S_t) = \frac{f}{\frac{(x-1)}{x}\hat{m}_{t+1}^B}$.[31] The fund return is expressed as $r_{t+1} = a(S_t) R_{t+1} - c(S_t) + \sigma_r \varepsilon_{t+1} = \frac{f}{\frac{(x-1)}{x}\hat{m}_{t+1}^B}\left(R_{t+1} - \hat{m}_{t+1}^B\right) + \sigma_r \varepsilon_{t+1}$.

[Figure 1 about here.]

Figure 1 displays the relation between fund returns and fund sizes. It shows a decreasing return to scale graphically for the model with both biases, where the exponent of cost function (x) is 1.883. True managerial ability is assumed to be at the mean level (22.03%) as estimated by SMM. The x-axis is the size of funds and the size of funds are corresponding to perceived abilities. Managers adjust their active shares according to their believes about managerial abilities and thus to fund sizes. The fund returns decreases slower than linear

[29] FOC of manager's optimization is $\hat{m}_{t+1}^B = C'(A_t) = bxA_t^{x-1}$.

[30] The equilibrium condition is
$S_t f = A_t \hat{m}_{t+1}^B - C(A_t) = A_t \hat{m}_{t+1}^B - bA_t^x = \left(\frac{\hat{m}_{t+1}^B}{bx}\right)^{\frac{1}{x-1}} \hat{m}_{t+1}^B - b\left(\frac{\hat{m}_{t+1}^B}{bx}\right)^{\frac{x}{x-1}} = b(x-1)\left(\frac{\hat{m}_{t+1}^B}{bx}\right)^{\frac{x}{x-1}}$.

[31] $a(S_t) = \frac{A_t}{S_t} = \frac{\left(\frac{\hat{m}_{t+1}^B}{bx}\right)^{\frac{1}{x-1}}}{\frac{b(x-1)\left(\frac{\hat{m}_{t+1}^B}{bx}\right)^{\frac{x}{x-1}}}{f}} = \frac{f}{\frac{(x-1)}{x}\hat{m}_{t+1}^B}$

and thus increase in managerial abilities allow the funds to manage larger size than the proportionally increased fund size. Managers decrease their active shares in fund sizes.

Simulation examples

[Figure 2 about here.]

In this subsection, I present simulation examples with figures. Figure 2 displays an impulse response of fund returns when there is one standard deviation positive shock in active return error at time 0. After time 0, active return errors are randomly drawn following the specified distributions. For the simulation, I employ the estimated distributions from SMM. Managerial ability shocks, non-return information signal errors follow the estimated distributions through all periods. Additional fund return errors are suppressed to zero, and initial managerial abilities are set by the estimated mean managerial ability. Panel A of figure 2 show time series graph of the mean fund returns for no-bias, overconfidence-only, overextrapolation-only, and both-bias model. The return at time 0 is positive for all models since all funds get a positive active return error shock. At time 0, investors update managerial abilities positively, but larger portion of positive fund returns are from active return errors than the portion which investors expect. Investors overestimate managerial abilities from fund returns and overinvest to the funds at time 1. But investors soon correct their estimates because they learn that the estimates are too large from the negative fund return at time 1. Overextrapolative agents responds more aggressively than overconfident or both biased agents because overconfident agents or both-biased agents put more weight on non-return information signal in updating believes. Overextrapolation amplifies the initial errors but overconfidence make agents less responsive to initial errors. Fund returns converge to zero after time 1 for no-bias model and overextrapolation-only model. Overextrapolative agents form positive persistent believes at time 0, but they soon correct their believes after experiencing negative returns. Fund returns of overconfidence-only and both-bias model do not converge to zero through all time. Investors less utilize information from negative returns at time 1 and, thus, they overestimate managerial abilities at time 2 and so forth.

The detailed mechanism can be shown by return sorts. I rank the funds by fund returns at time 0 and track the fund returns of each return quintile portfolios. The figure 2 B is the lowest quintile returns for the four models. The lowest return quintile funds get largest negative ability shocks and positive one standard deviation active return error shocks. The agents update managerial abilities positively since the net effects of ability shocks and active return errors are still positive. Overconfident agents face difficulties to correct estimation errors once estimation errors are created. Fund returns do not converge to zero through all time and magnitudes are quite large. If investors are overconfident on their non-return signals, they put too much weight on their non-return signals about managerial ability changes and ignore negative returns at time 1.

Convexity

[Figure 3 about here.]

Figure 3 A shows that return-flow relationship from the simulation. The parameter values are assumed from the SMM results. Figure 3 A is simulated from the rational agent case. In the model, convexity of return-flow relationship comes from convex trading costs and choice of active shares. Fund managers change proportion of actively managed funds as their perceived abilities change. A marginal increase in a manager's ability enlarges the break-even size greater than linear. Thus, a convexity of flow depends on an exponent of a cost function.

Sensitivity and age

Figure 3 B compares return-flow relationships at different time horizons. The blue solid line represents the relationship at time 1 and yellow dashed line shows the relationship at time 20. Sensitivity of flows to return depends on two sources of uncertainties: initial uncertainties and managers' ability change uncertainties. In initial period, investors are more uncertain about fund managers' abilities. An initial uncertainty decreases and converges to a stationary uncertainty. However, investors never fully know managers' abilities because managers' abilities vary over times. Investors will respond more aggressively due to large uncertainties for young funds and less aggressively for old funds. We can see that the slope of solid line is steeper than the slope of dashed line. The return-flow curve converges to the curve at the stationary estimation error. Unlike Berk and Green (2004), converged return-flow curve still has a positive slope.

4. Data and moment selection

I use mutual fund data from the Center for Research in Security Prices (CRSP). Following Linnainmaa (2013), I only include actively managed funds that invest in U.S. common stocks. CRSP provide an unified fund objective code, i.e. CRSP objective code, which incorporates many other databases. I include the funds of which an objective code starts with "ED" except "EDS". I combine the information of multiple share classes of the same fund using MFLINKS of Wharton Research Data Services. Information of mutual funds are computed in a value weighted way using a net asset value of each share classes. I also use a text analysis to exclude index funds and index leverage funds. The sample data of mutual funds start from January 1962 and end at December 2012. The length of data is around 50 years. I exclude mutual funds with smaller assets than $5 million in December 2006 dollars.[32] The fund size restriction is related to an incubation bias. Once they pass the size threshold, I keep the funds until they disappear from CRSP. I utilize mutual fund data from 1967 to 2012 because the risk-adjusted fund returns require at least three year previous observations.

There are total 58,035 fund-year observations. After only including actively managed funds, a number of fund-year observations decreases to 51,912. The total number of actively managed funds over the sample periods is 3,594. Over the sample periods, the minimum number of existing funds cross-sectionally is 142 and the maximum number is 2,719. 1,452 funds disappear during the sample periods. 2,142 funds survive at year 2012.

Following a common practice of literature, I use the Carhart (1997)'s four factor model to adjust the fund return risk. The risk adjusted return is

$$r^e_{i,t} = r_{i,t} - r_{f,t} - \beta_{m,t}\left(r_{m,t} - r_{f,t}\right) - \beta_{SMB,t}SMB_t - \beta_{HML,t}HML_t - \beta_{MOM,t}MOM_t.$$

where $\beta_{m,t}$, $\beta_{SMB,t}$,$\beta_{HML,t}$,$\beta_{MOM,t}$ are the estimates of three-year rolling window regression. For a return predictability regression, factor loadings are from previous three-year fund returns. Short-lived funds are excluded because risk-adjusted returns are not available. As a robustness check, I fill out short-lived fund risk-adjusted returns using stock holding information. From stock holding information, I construct past three year portfolio return of the same stock holding weights. I run regression on the constructed portfolio returns and back out the factor loadings. I compute the risk-adjusted return using the backed-out factor loadings and mutual fund returns.

Moments

[Table 1 about here.]

[Table 2 about here.]

To confirm well-known stylized facts and use moments from mutual fund dynamics, I summarize mutual fund returns and flows, and regression results. For value-weighted and equal-weighted risk-adjusted fund returns, I first compute value-weighted and equal-weighted after-fee monthly fund returns from CRSP fund returns. I run four-factor regressions for

[32] I follow Linnainmaa (2013) for this criterion.

entire sample years and compute risk-adjusted returns. As a robustness check, I run three-year window rolling regressions and get similar results.[33]

I employ Fama-MacBeth regression for estimation of fund moments. Fama-MacBeth regression is not an efficient methodology for panel data since they aggregate cross-sectional information and only use time series variation. Inefficiency is more severe for mutual fund research because mutual fund data has much larger cross-section than time-series. In other words, Fama-MacBeth is a conservative methodology. I apply three year-lag Newey-West adjustment for covariance across moments. It accounts for serial correlation between moments themselves and other moments as well. I choose three year lag since the rolling window is three year horizon. Table 1 panel A shows that the aggregate value weighted return is - 70.7 basis points and is significantly negative at the 1% level. The equal weighted return is -21.2 bp and is insignificantly negative at the 10% level.[34]

In the model, I do not consider additional costs such as brokerage fee and front-end and back-end load like other research. French (2008) argues that the additional cost is around 1.5%. Negative 71 basis points is not yet small, when we consider a total size of mutual fund industry. Taking into account the additional cost, the level of underperformance is about 2% annually.

Table 1 panel B shows cross-sectional fund returns. Fund returns are risk-adjusted using four-factor rolling regression with three-year window. Because short-lived funds may occur a missing sample bias, I use backed-out factor loadings from stock holding information and compute risk-adjusted returns if returns cannot be adjusted through rolling regression.[35] A 25th percentile return is -444 basis point and is quite stable over times. It is significant at the 1% level. A median return is -52.8 basis point and is less than zero. It is significant at the 5% level. A 75th percentile return is 347 basis point and is significant at the 1% level. The standard deviation of fund returns is 8.96 percent and is significant at the 1% level. A fund return cross-section is distributed similarly through all times.

Table 1 panel D summarizes fund flow distributions. A 75th percentile flow is 19.9% and is significant at the 1% level, a 50th percentile flow is -2.18% and is insignificant at the 10% level, and a 25th percentile flow is -12.1% and is significant at the 1% level. Flow distributions appear to be stable over times.

Table 2 panel A displays a return-flow regression. I regress a fractional fund flow of this year on a lagged fund return. As in previous literature, I winsorize fund flows at the

[33]As a robustness check, I also test a different order to make risk-adjusted aggregate returns. I first run regression to adjust risk for individual fund returns and aggregate risk-adjusted fund returns.

[34]Instead of Fama-French three factors, I also use Cremers' three factors which fix downward bias in returns of small and value category.

[35]I require at least 30 months of returns for a 36 month window.

5% level. As fund returns increase, percentage fund flows also increase. The coefficient on fund returns is 1.64, which implies that 1% higher fund returns induce 1.64% more percentage flows. It is significantly positive at the 1% level. Fund flows respond convexly to fund returns. The coefficient on the fund return square is positive and is significant at the 1% level. One percentage higher fund returns lead 1.59% higher sensitivity in fund flows. The results confirm statistically significant convex return-flow relationships over the selected sample periods. The interaction term between ages of funds and fund returns is negative and is significant at the 1% level. This is a unique feature of the nonstationary learning model. The sensitivity of flows decreases throughout. Table 2 panel B includes size quintile dummy variables to test the heterogeneous responses to fund returns for funds with different sizes. Interaction terms between fund returns and dummy size quintiles show that percentage flows of larger funds are less sensitive to their lagged fund returns.

[Table 3 about here.]

Table 3 panel A shows a fund return persistence regression. Since the risk-adjusted return is from three-year rolling regression, this procedure may give a mechanical persistence. I use previous three year fund returns to estimate betas of risk factors and compute risk-adjusted returns using these betas. I regress a risk-adjusted return on a one-year lagged risk-adjusted return. The result shows that 10.4% of abnormal returns remain in a subsequent year. The coefficient is significant at the 1% level. I also include interaction terms between fund returns, and highest and lowest previous year return decile groups in panel B. It shows that a return persistence is weaker for extreme decile groups. We can interpret this result as normal funds other than worst or best performing mutual funds drive fund return persistence contrasting to Berk and Tonks (2007). One-year horizon fund return continuation exists regardless of previous return performances. Panel C includes interaction terms between fund returns and size quintile dummies. The result indicates that return persistence also does not depend on fund sizes.

[Table 4 about here.]

5. Estimation

I utilize the simulated method of moments (SMM) as a structural estimation methodology. There are 11 parameters to be estimated for the the both-bias model: mean ability of mutual fund manager, $\bar{m}$, standard deviation of manager's ability change shock, σ_m, standard deviation of active return error, σ_e, non-return signal precision, θ, mean reversion of managers' ability shock, ρ, exponent of fund size for cost function, x, initial uncertainty of manager's ability, σ_{m0}, standard deviation of additional return error, σ_r, survivorship threshold, a_{exit}, perceived biased non-return signal precision, θ_b and perceived mean reversion, ρ_b.

The model is flexible to explain many stylized facts for mutual funds. I employ mainly three stylized facts: negative aggregate return, return persistence, convexity in return-flow relationship. I pick 15 moments from cross-sectional distributions and stylized facts. An equal-weighted fund return and a value-weighted fund return are related to the negative aggregate return. $\beta_r^{ret-flow}$, $\beta_{r^2}^{ret-flow}$, and $\beta_{age\times r}^{ret-flow}$ are coefficients of regressors from the return-flow regression. I regress fund flows on previous one-year returns, return square terms and interaction terms between ages of fund and fund returns. Size 80th perc./size 50th perc. and size 50th perc./size 20th perc. are relative size distributions, i.e. 80th percentile size over 50th percentile size and 50th percentile size over 20th percentile size. Because the model does not have any implication for absolute sizes of funds, I use relative size distributions as moments. $\beta_r^{ret\,pred}$, $\beta_{r\times I_{bottom}}^{ret\,pred}$, and $\beta_{r\times I_{top}}^{ret\,pred}$ are coefficients of regressors from the return persistence regression. I regress one-year risk-adjusted returns on previous one-year risk-adjusted returns and interaction terms between fund returns, and bottom and top return decile dummies. Probability of successive top deciles and probability of successive bottom deciles are the probabilities of top and bottom return decile funds at time t being in the same decile at time t+1. All returns are adjusted following Carhart's four factors. Flow 75th percentile - flow 25th percentile is a difference between 75th percentile and 25th percentile percentage flows. Disappearing probability is a disappearing fraction of funds at each year. Return prior to disappear is a one-year return before funds exit from the industry. Overall, the model tries to match three stylized facts and cross-sectional distributions of returns, sizes, flows and survivorships, which cover general dynamics of mutual funds. Cross-sectional distributions include first and second moments except the second moment of fund returns and the first moment of fund flows. Risk-adjusted returns contain estimation errors because they are estimated from rolling regressions over 36 month windows. Noises added from estimations enlarge standard deviations of fund returns and misidentify return distributions. I exclude the standard deviation of returns for this reason. The mean fund flow is excluded as well, because the model do not rationalize a steady increase in aggregate fund sizes over forty years. Since the number of moments is greater than the number of parameters, SMM is over-identified. Over-identifying restriction will give goodness of fit of the model, which is equivalent to out of sample tests.

I estimate the moment weight matrix directly from the data using Fama-MacBeth method.

If moments are coefficients of regressions, I first run a regression cross-sectionally for each year observation. If moments are distributional information of cross-section, I compute distributional information for each year. After gathering moments for each year observation, I apply Newey-West covariance adjustment to the vector of moments with three-year lags. I use Laplace type estimator (LTE) method following Chernozhukov and Hong (2003), a quasi-bayesian methodology, as a SMM methodology. This method is known as efficient way to avoid a local maxima problem. In each simulation, I generate 20,000 funds over 45 years, which is a sample observation year. I repeat 10,000 times to complete estimations.

Identification strategy

I choose the moments for economic reasons, but they also pin down the parameters well. In this subsection, I will explain how the moments can identify parameters. Mean ability of mutual fund manager ($\bar{m}$) is closely related to many moments. The return moments co-determine $\bar{m}$ and other parameters even though the observed return is after the adverse fund size effect. $\bar{m}$ is also relevant to size and flow distributions. Disappearing probability and disappearing return also depend on $\bar{m}$. Decrease in $\bar{m}$ increases the probability of hitting the survivorship thresholds.

Standard deviation of additional return error (σ_r) is pinned down by fund flow cross-section, return-flow relationship, and return persistence moments. Because additional return errors add return variance which cannot be explained solely by learning part, it changes flow or return sensitivity to return. To generate enough return variance without the additional return error, learning also magnifies flow volatility. In order to fit both flow and return distribution, additional error is necessary. In my model, investors can observe this error and understand that it is irrelevant to managers' ability.

Standard deviation of manager's ability change shock (σ_m) can be mainly pinned down by return-flow relation and size distribution. It also relates to return and flow distribution. Investors respond to historical return by allocating flows because the managers' ability changes over time. Thus, return-flow relation is closely related to σ_m. Size distribution also tells about the magnitude of managers' ability distribution at each cross-section.

Standard deviation of active return error (σ_e) is related to the accuracy of learning. With larger σ_e, investors face more difficulty to learn about manager's ability. In the non-stationary setting, it is related to speed of convergence to stationary learning distribution. Interaction term between return and age in return flow regression has a relationship to the speed. Thus, σ_e can be pinned down by the return-flow relationship and flow distribution.

The non-return signal precision (θ) is specifically involved with return-flow regression.

With higher non-return signal accuracy, investors rely more on non-return signal and respond less strongly to return. For overconfidence case, it also related to return predictability regression. The return persistence is depending on persistent mistake by investors and the degree of mistake can be lessened by accurate non-return signal. Survivorship thresholds (a_{exit}) is determined solely by disappearing probability.

Mean reversion of managers' ability shock (ρ) is pinned down by flow distribution and return-flow relationship. If the mean reversion becomes stronger but return variance does not vary, flow becomes more volatile, and investors will less allocate flow to the same amount of return.

Exponent of fund size for cost function (x) is related to size distribution and return-flow relationship. Larger x results in less dispersed size distribution because small increase in size incurs large cost. For high x, investors allocate flows less convexly because marginal increase in cost is larger than linear for marginal increase in size.

Initial uncertainty of manager's ability (σ_{m0}) is identified by interaction term between fund return and age of return-flow regression. The information amount acquired by observing fund return depends on the initial uncertainty. If initial uncertainty is high, investors will respond more to fund return because relative information content of fund return is high.

Moments for negative aggregated return and return persistence are equivalent to out of sample test. For unbiased case, these moments are not relevant, but for biased case, they identify perceived biased non-return signal precision (θ_b) and perceived biased mean reversion parameter (ρ_b) .

Estimation results

[Table 5 about here.]

[Table 6 about here.]

This section analyzes the model's fit to the data. The moments are selected to describe full mutual fund dynamics including well-known stylized facts. Moments are corresponding to one of categories: aggregate return anomaly, return predictability, return-flow relationship, size, flow and return distributions, and disappearing return and probability. The first two columns of table 6 display empirical estimation of 15 moments along with the standard errors. The third column of table 6 displays the results from SMM estimation of the model with both biases where minimizes the SMM objective function. The fourth column of table 6 presents t-statistics of difference between empirical estimation and simulated results. The overall fit of moments appears reasonable. Most individual moments are not statistically significant at the 10% level. The model with both information biases can match moments from return persistence, and aggregated return successfully. The probability of successive bottom deciles is significantly different from the data moment at the 5% level. The model generates 4% smaller probability than the data moment. The return-flow relationship coefficients $\beta_{r}^{ret-flow}$ and $\beta_{r^2}^{ret-flow}$ are significantly different from the data moments at the 10% level and at the 5% level respectively. The response to fund returns in the simulation is more convex and sensitive than the empirical response. The percentage errors of economic magnitudes for these moments are 27% and 73%. The interaction term between a fund return and a top decile dummy in the return predictability regression gives opposite sign and is statistically significant at the 10% level. The other moments show less than 20% difference in their economic magnitudes.

Notably, the model matches negative aggregate return, and return predictability. The rational model of Berk and Green (2004) cannot generate these anomalies.[36] The model successfully matches size, flow and survivorship moments as well. The last row of table 6 provide $\chi^2(4)$ statistics of 5.9963, which gives p-value of 0.1994. The model is not significantly different from data moments at the 10% level. The model abstracts many elements from the real world, and therefore the model cannot fully explain whole dynamics at the same time. But the model successfully explain the selected moments which are selected not for fitting purpose but for economical reasons. This success does not imply that the model is right. But at least, it is useful as a benchmark model for future research. The pattern of mutual fund dynamics is not just random as seen that the model can connect dynamics into a single framework. While the data has a lot of noises and the model has missing components, the model gives plausible fit to the data and explains most important mechanisms in mutual fund dynamics.

Parameter Estimates

Table 5 displays parameter estimates and standard errors of a model that generates the simulation with best fit to the data. The standard errors of 11 parameters are relatively small to their values. The set of moments identify the model parameters successfully. The estimates of manager's mean ability ($\bar{m}$) is 22.03%. It implies that investors make 22.03% marginal return for the first dollar of their investments. It does not mean that funds generate positive fund returns on average. The average return of funds depends on managerial abilities, active shares, and fund sizes. A poor performance is resulted from a larger size of funds relative to their managerial abilities. The median marginal ability (22.03%) corresponds to the fund size

[36]Table A1 shows the Berk and Green estimation with the selected moments in the appendix.

of 215 million dollars that this manager afford to run without any excess returns after fees. The marginal ability of the manager who manages one billion dollar is 45.28% and that of 100 million dollar fund is 15.38%. Later, I will discuss about the optimal size distribution of funds as a first-best solution. The estimate of mean fund return ($\bar{m}$) is significantly different from zero at the 1% level. The standard deviation of annual ability shock (σ_m) is 5.69%. A fund manager receives an ability shock of 5.69% annually. The true mean reversion parameter (ρ) determines persistence of the ability shock. The estimated mean reversion parameter is 0.0673 and is statistically significant at the 1% level. The half-life of the ability shock is 9.95 years. The ability shock is quite persistent and the magnitude is smaller, but comparable to the long-run uncertainty component of Bansal and Yaron (2004)[37].

The estimated standard deviation of active return error is 17.86%. Berk and Green (2004) set the standard deviation of active return at 20% considering a historical level of portfolio volatilities. The standard deviation is smaller than the assumption of Berk and Green (2004), and it is within a reasonable range considering that the average individual stock standard deviation level is 59%. The actively managed fund assets are mostly concentrated portfolios after excluding portions for tracking the benchmarks. In this sense, the estimate of 17.86% is quite large, but is still acceptable.

The estimated precision of non-return signal (θ) is 0.3232. This parameter do not have a comparable benchmark because the non-return signal process is latent. In reality, investors receive non-return signals from star system of morningstar, major news providers, advertisements, brochures, award winning information and many other information sources. The non-return signal delivers quite accurate additional information to investors in the sense that the accuracy of the non-return signal for managerial ability change is 32.3%.

The estimated exponent of cost function (x) is 1.88327. The estimate is smaller than 2, which is the assumption of Berk and Green (2004). It implies that costs of funds increases slower than quadratic. [38] The trading cost is only associated with actively invested amounts of funds. If estimated managerial abilities double, funds can increase sizes as 2.19 times, larger than double considering that managers decrease active shares at the same time.

The perceived initial standard deviation of managerial abilities (σ_{m0}) is 0.1933. In other

[37] The half life of the long-run risk shock is 32.66 years.

[38] Chen et al. (2004) show the abstract relationship of return and log size of fund by ordinary least square. If the relationship is quadratic, then the increase in size decreases fund return in a square root due to decreasing active share. The coefficient of Chen et al. (2004) is -0.0036 which is equivalent to 1.5036 in the model.

$$-\text{size}^{0.0036} = -\exp(0.0036\,\log(size)) \simeq \text{-}(1 + 0.0036\log(size))\text{=-1-0.0036}\log(size).$$

The estimate of 1.88327 is larger than the estimate of 1.5036 in Chen et al. (2004). But the direct comparison to Chen et al. (2004) is inappropriate because Chen et al. (2004) find the relationship after the adverse size effect. In later section, I will test the size and return relationship in a reduced form as an out of sample test.

words, agents believe that standard deviation of estimation errors is 19.33% at the beginning of funds. The perceived standard deviation of stationary estimation errors is 4.33%. The estimation is significantly different from the perceived stationary estimation error at the 1% level. We can conclude that investors initially have more uncertain prior on the manager's ability than stationary one. The perceived precision of the estimation enhances over times, and converges to the stationary precision.[39] When the funds are established, the perceived uncertainty is 4.46 times larger than the stationary uncertainty. As in Berk and Green (2004) [40], I also find that initially investors have less information about funds and learn more as time goes by. The estimated standard deviation of additional return error (σ_r) is 2.31%. Investors understand that 2.31% of fund returns comes from the random events uncorrelated to manager's ability. The variance of additional return errors is the part of total return variance irrelevant to the manager's ability such as the abnormal returns from some macro events or inevitable trading costs due to sudden investors' redemption. The estimated survivorship threshold (a_{exit}) is 0.00191. Funds disappear when investor's belief about fund manager's current ability is lower than 0.191%. The exit threshold of Berk and Green (2004) is set up as 3% with assumption that funds has a fixed running cost in each period. I also do not set up the entry/exit problem of funds in the model. The threshold abstracts the entry/exit problem in the reduced form. If funds have initial entry costs, then funds do not exit even though funds occur fixed costs and lose reservation opportunities. The exit threshold can be lower than 3% and even can be negative in this sense.

The estimates of information processing parameters (θ_b, ρ_b) are significantly different from their true parameters (θ, ρ). The perceived precision of the non-return information signal (θ_b) is 0.7554 and is significantly different from the true estimate of 0.3232 and the maximum precision of 1.0. It implies that agents think their non-return signals are 2.34 times more accurate than the actual precisions. This overconfidence produces the negative aggregate return and positive return predictability as described earlier. The estimate of perceived mean reversion in manager's ability (ρ_b) is 0.0519, implying that agents overextrapolate the managerial ability shock. The perceived mean reversion parameter (0.0519) is not significantly smaller than the true mean reversion parameter (0.06733) at the 10% level. Both information processing biases are necessary to match magnitudes of return predictability and negative aggregate return while matching other mutual fund dynamic moments. The difference of perceived errors between biased agents and rational agents is 13.4% of variance in true managers' abilities.

[39] Investors do not completely learn manager's ability because the managerial ability gets a shock at each period.

[40] In Berk and Green (2004), investors learn about fixed manager's ability which is unknown to investors.

The magnitude of overconfidence bias is comparable to the level of overconfidence bias in Alti and Tetlock (2012). Alti and Tetlock (2012) find the degree of overconfidence bias in the estimation of firm's productivity using a structural model in the same fashion as this paper. They figure that the difference in the biased and rational agents' perceived errors is 19% of the magnitude of the volatility in true productivity. As in Alti and Tetlock (2012), I compare the magnitude of overconfidence bias in the model to direct measures of expectations from surveys and forecasts. Table 2 of Alti and Tetlock (2012) shows that perceived confidence intervals of three surveys are narrower than realized confidence intervals. In other words, participants of the survey believe that their predictions are more accurate than realization. I find that the 80% confidence interval implied by the overconfidence is correct 50.3% of the time. Investor's 90% and 98% confidence intervals are correct 61.7% and 78.3% of the time respectively. The degrees of overconfidence is less than the survey results, where 80%, 90% and 98% confidence intervals are right 33%, 50% and 54% of the time. The survey participants are including executives, money mangers, and HBS students. The agents in the model shows less overconfident than quite general survey participants. It rationalize why the biases are difficult to be corrected over times.

The economic magnitude of overextrapolation is harder to compare with the estimates from the surveys or lab experiments than overconfidence. Overextrapolation is more subject to a specific stochastic process. The difference of ρ and ρ_b is small at 1.5%, but it helps to match the return predictability and negative aggregate return.

Alternative Model Estimates

In this section, I analyze the estimates of the models with a single bias or without a bias. The column 5 and 6 of table 6 display the moment estimation from the no bias model. As predicted, the model fails to match the negative aggregate return and the return predictability. The value-weighted mean return is almost zero and significantly different from the data moment at the 5% level. The return predictability does not exist on the contrary to the data. The return prior to disappear is lower than the data and rejected at the 1% level. The χ^2 statistics is 25.046 and the p-value (0.0003) is very close to zero. The model is significantly rejected at the 1% level.

The columns 7 and 8 of table 6 show the estimation of 15 moments along with the standard errors. The overconfidence only model is rejected at 1% significance level. We can compare the both bias case and overconfidence only case with χ^2 values. The difference of χ^2 values is 21.6 - 6 = 15.6 which suggest the additional bias improves a fit to the data. The overconfidence only model displays a difficulty in matching the negative value weighted return, disappearing return, size and flow distribution compared to both bias model. But the model successfully matches return predictability moments. The model requires larger magnitude of overconfidence to match the negative aggregate return, but would worsen the fit of the other moments in that case. Another comparison criteria is the plausibility of the estimated parameters. The columns 5 and 6 of table 5 show the estimates and standard error of parameters for the overconfidence only model. The estimated active return error (29.2%) is too large considering the average individual stock standard deviation level (59%). The estimated size exponent x of cost function (1.4729) is quite smaller than the estimate (1.8833) of the both bias model. The funds can increase the size as 4.33 times when the managerial ability doubles.

The last two columns of table 6 presents 15 moments estimation from the overextrapolation only model. The model suffers to match the return predictability moments. The probability of successive bottom deciles is significantly different from the empirics at the 1% level and interaction term of return and bottom decile dummy in predictability regression is also rejected at the 1% level. The magnitude of return predictability is weaker than the data. The χ^2 of the model is 13.923 and the p-value is 0.016. The model is rejected at the 5% level, but not rejected at the 1% level. The last two columns of table 5 display the parameter of the overextrapolation only model. The estimated active return error (25.9%) is also large like the estimate of the overconfidence only model. The estimate of size exponent x of cost function (1.4728) is same as the estimate of the overconfidence only model.

The estimates of mean abilities vary a lot across models. The estimate of no-bias model is only 5.4%, but the estimates of rest models are greater than 20%. One possibility is that biases make perceived managerial abilities more often hit the exit thresholds and raise the probability of exit. To lower biased exit probabilities, models with biases require higher mean managerial abilities than model with no-bias. The exit thresholds also have large variation across models. The exit thresholds are much smaller than the mean levels of abilities for models with biases. It makes that the exit thresholds are less well identified and are less important for entire model fit.

With the estimated parameter set, the overconfidence only model cannot produce enough negative aggregate return, but the overextrapolation only model can yield observed negative return. As shown in the appendix, perceived manager's ability is normally distributed around the true manager's ability in the overconfidence only model. In the overextrapolation only model, the estimated manager's ability is normally distributed, but has a mean of $(1 + k) \times (m_t - \bar{m}) + \bar{m}$, where m_t is the true manager's ability, $\bar{m}$ is the mean ability, and k is a positive constant. Estimation errors are skewed conditional on the true managerial ability. There exist more positive estimation errors for the higher managerial ability. It leads to enough negative aggregate return because the weight is size. On the contrary, in the overconfidence only model, the size difference around the true ability is small, and much of positive estimation errors and negative estimation errors cancel out in the value weighted way.

The overconfidence can generate moderate return predictability. The overconfident agents continuously ignore the return history and the estimation errors are not corrected in a short term. The overextrapolative agents think that the managerial shock is more persistent than the actual and they allocate excessive money into the funds with positive return. In the next period, they continuously allocate the excessive money into the same funds due to persistent belief, but the overreaction to the following negative fund return cancels out a part of persistent belief effect. As a result, overextrapolation by itself has difficulty to produce enough amount of return persistence with the estimated parameter set.

[Table 7 about here.]

[Table 8 about here.]

I also estimate the parameters of Berk and Green (2004) using the selected moments using SMM. Table 7 and table 8 show the parameter estimates and the moment estimation from the SMM. The χ^2 statistics is 62.0896 and the p-value (0.0000) is almost zero. The model cannot match the negative aggregate return, return predictability and the survivorship moments. There is no negative aggregate return and return predictability in the rational model. The probability of successive top deciles and the probability of successive bottom deciles are significantly smaller than the data empirics at the 1 % level. The model fits the size distribution due to small covariance related to size moments. The estimate of size exponent x of cost function (1.13) is much smaller than the estimates of the models considered in this paper and 2 of Berk and Green (2004) assumption. Compared to the no bias model in this paper, Berk and Green (2004) restrain the time-varying ability and the non-return information channel. The difference of χ^2 is 62.09 - 25.05 = 37.04, which is quite large. We can see the importance of time-varying ability and the non-return information channel to explain the data.

6. Out of Sample Tests

[Figure 4 about here.]

In this section, I will conduct out of sample tests using the simulated data. The simulated data are generated by the parameter estimates of both bias model. Figure 4 presents the return paths of time 0 return sorted groups from actual data and simulated data. I construct return quintile portfolios at year 0 and tracks the portfolio returns up to year 5. Sub-figure A is from actual data and sub-figure B is from simulated data. In sub-figure A, the portfolio returns are quite dispersed at the sorting year. The spreads are diminished a lot in year 1, and are gone from year 2. In sub-figure 2, the simulated return patterns seemingly resemble the actual return patterns, while the return difference among return quintiles are smaller at year 0 and a part of return remains until year 5 in the highest return quintile group. I do not take a statistical comparison here, but the return persistent patterns give some sense that the model has a explanatory power in wide ranges of return persistence.

[Table 9 about here.]

In Berk and Green (2004), flows do not predict returns. Thus, if there are no post-allocation return differences, it supports their equilibrating through fund flows arguments. In table 9, I compare the returns of flow quintile groups between actual data and simulated data. The patterns look similar in a way that mean returns of each quintiles are quite similar and flows and returns are positively correlated at year 0, but pattens at year 1 is not trivial. At year1, lowest flow quintile has insignificantly positive returns and returns of other quintiles are significantly negative but indifferent among these quintiles. These patterns support partially equilibrating arguments of Berk and Green (2004). In my model, flows are mixed response of rational force and behavioral force. One may think that flows are dominated by misallocation due to biases and are more correlated to ex-post performance. But simulation results show that flows driven by rational response weaken correlation between flows and returns as in the data. The simulation also presents the patterns that only lowest flow quintile has a positive return and others have a negative but not in sorted way. In total, 7 out of 10 differences are not significant statistically at the 10% level.

[Table 10 about here.]

Chen et al. (2004) and Cremers and Petajisto (2009) are related to mutual fund return predictability by the fund size and active share of fund investments respectively. Chen et al. (2004) find that a mutual fund return decreases in a fund size. They run the following regression to show the size and return relationship:

$$r_{i,t+1} = \beta \log(\text{size}_{i,t}) + \varepsilon_{i,t+1}.$$

They include other control variables, but the controls are not included in the estimates from the simulated dataset. Chen et al. (2004) discovers that the negative relationship between the fund size and fund return. The coefficient estimates of log size from Chen et al. (2004) is -0.00336. This number is difficult to be compared with model parameter, since the estimated relationship is after the equilibrium effect. For this reason, I generate the simulated dataset which is after the equilibrium effect, and see the relationship between fund size and ex-post fund return. The estimate is -0.00381 and statistically significant at the 1% level. The economic magnitude are very close as the percentage difference is only 13%. However, the estimate of no bias model is significantly different from empirical estimates at the 1% level. No bias model do not generate the negative size-return relationship as predicted. The rational model does not allow any return predictability because the difference in expected fund return will be exploited by the rational agents.

Cremers and Petajisto (2009) figure out that a mutual fund return increases in the actively managed share of total fund investments. They regress the active share on the next period fund return:

$$r_{i,t+1} = \beta \text{active share}_{i,t} + \varepsilon_{i,t+1}.$$

The estimates of active share in Cremers and Petajisto (2009) is 0.0722 and statistically significant at the 1% level. The estimates of both bias model is 0.06996 and significant at the 1% level. The difference between the estimates is not rejected at the 10% level. The economic magnitude of difference is 0.00224 and 3% in percentage error term even though the active share measure of Cremers and Petajisto (2009) is a proxy measure of actual active share. As a same reason of size-return relationship, the rational model predict no relationship between the active share and the return. The result shows that relationship from the model is significantly different from the relationship from the data at the 1% level.

Frazzini and Lamont (2008) find the dumb money flows in mutual funds. The fund flows are allocated in the wrong place in a sense that investors could have done better if they allocate the money proportional to the previous allocation. This argument does not take into account the decreasing returns to scale or equilibrium effect. I follow the procedure of Frazzini and Lamont (2008) to compute the counterfactual value-weighted return from the simulated dataset. I also replicate their empirics to match the sample periods of moment empirics and to use the risk-adjusted return as a fund return measure. I find that the counterfactual value-weighted return is larger than the value-weighted return, but it does not necessarily imply that investors worsen the investment return by reallocating actively since we ignores the equilibrium effect. The replicated estimate has smaller magnitude, but is in the same sign. The magnitude of estimates of simulation and actual data is around -1 basis point and the difference is only 1.1 basis point.

From the two predictability regressions and dumb money calculation, we can witness the plausibility of the simulated dataset. The estimates align with the results of the literature and are even very close to their estimates economically. The out of sample tests could give us an extra credit to the model and the sense that a success of moment matching is not from the moment mining.

7. Optimal size of funds

In this subsection, I run a simulation to find the correct size for the mutual fund industry. Since the size is not absolute, I find the percentage size difference between the both bias model and the no-bias model. In the no-bias model, I suppress the information biases to be zero. When I calculate the choice of agents without biases, I assume that manager's have the same previous estimation and information as the biased agent in each period. By comparing two simulated sets, I find the current fund size is relatively large to its managers' capability to manage the observed fund size. The size of misallocated funds is 8.58% of total fund size. Misallocation is the sum of absolute difference between allocation of biased agents and unbiased agents. The optimal aggregate fund size allocated by unbiased agents is about 4.63% less than the one allocated by biased agents. Since the coefficient of cost function is not exactly 2, it is difficult to conclude that the decreasing 4.63% exactly implies that the aggregate fund return will be zero. At least decreasing 4.63% is required to have a aggregate return close to zero.

The inefficiency due to misallocation has an impact on the stock market. The oversize of active funds incurs over-concentration in US equities among asset classes. The fraction of stock market capitalization owned by active mutual funds [41] at the end of year 2012 is 16.9%. The active funds have 4.63% larger sizes than the optimal fund sizes, and thus overinvestment to stock market driven by active mutual funds is $4.63\% \times 16.9\% = 0.78\%$, 0.78% of stock market capitalization. It implies that 0.78% of equity values are overvalued due to behavioral biases of mutual fund investors.

8. Conclusion

In this paper, I develop a model to explain mutual fund dynamics. The model features a behaviorally biased learning about the time-varying managerial ability and decreasing returns to scale. Investors equilibrate expected fund returns through fund flows in a biased

[41] The fraction of stock market cap owned by active mutual fund is the sum of funds' AUM divided by total market cap of CRSP US stocks.

way. The information processing biases, overconfidence and overextrapolation, cause the negative aggregate return and return persistence. I structurally estimate a model by the simulated method of moments. The model with both biases shows better fit than other alternative models. The model with both biases is not rejected at the 10% level and matches well most of 15 moments. The model successfully matches negative aggregate return, return persistence, size, flow and survivorship moments. The model with a single bias or no bias cannot match negative aggregate return and return persistence simultaneously. The model suggest that active management displays decreasing returns to scale, and an equilibrating forces exists by quasi-rational investors. I also conclude that fund managers have ability, but the overinvestment to the funds results the negative fund return.

We need to be careful to make a prediction from the parameter estimates of this paper. The model parameter estimates are from in-sample estimation. One might interpret the results in a way that the negative performance will continue to exist as long as the behavioral biases and the information asymmetry exist. It does not imply that the behavioral biases will continue to exist at the same level in the future. It is possible that agents may realize the behavioral biases and correct them. We may implement a policy to prevent overinvestment later and reduce the effect of behavioral biases.

As a future research, one could consider heterogeneity between fund manager and investors. Fund managers are less subject to behavioral biases, but investors have larger degree of biases. With the heterogeneity, one could also consider a moral hazard of fund managers. Fund managers have more information about their skills and incentive to hide their skills. These consideration can enrich the predictions of mutual fund dynamics. One might extend the analysis for the hedge fund dynamics. The fee structure assumption of the model is specific to mutual funds, but we can generalize the model to account for all active management industry including hedge funds.

References

Alpert, M. and H. Raiffa (1982). A Progress Report on the Training of Probability Assessors. In D. Kahneman, P. Slovic, and A. Tversky (Eds.), *Judgment Under Uncertainty: Heuristics and Biases*, pp. 294–305. Cambridge University Press.

Alti, A. and P. C. Tetlock (2012). Biased Beliefs, Asset Prices, and Investment: A Structural Approach. *SSRN Electronic Journal*.

Bailey, W., A. Kumar, and D. Ng (2011). Behavioral biases of mutual fund investors. *Journal of Financial Economics 102*(1), 1–27.

Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *The Journal of Finance 59*(4), 1481–1509.

Barber, B. M. and T. Odean (2001). Boys Will be Boys: Gender, Overconfidence, and Common Stock Investment. *The Quarterly Journal of Economics 116*(1), 261–292.

Barberis, N., A. Shleifer, and R. Vishny (1998). A model of investor sentiment. *Journal of Financial Economics 49*(3), 307–343.

Ben-David, I., J. R. Graham, and C. R. Harvey (2013). Managerial Miscalibration. *The Quarterly Journal of Economics*.

Benartzi, S. (2001). Excessive extrapolation and the allocation of 401 (k) accounts to company stock. *The Journal of Finance LVI*(5), 1747–1764.

Berk, J. and J. V. Binsbergen (2012). Measuring Skill in the Mutual Fund Industry. *SSRN Electronic Journal*.

Berk, J. and I. Tonks (2007). Return persistence and fund flows in the worst performing mutual funds. *SSRN Electronic Journal* (23).

Berk, J. B. and R. C. Green (2004). Mutual Fund Flows and Performance in Rational Markets. *Journal of Political Economy 112*(6), 1269–1295.

Bollen, N. P. B. and J. A. Busse (2004). Short-Term Persistence in Mutual Fund Performance. *Review of Financial Studies 18*(2), 569–597.

Carhart, M. (1997). On persistence in mutual fund performance. *The Journal of finance LII*(1).

Chen, J., H. Hong, M. Huang, and J. D. Kubik (2004). Does Fund Size Erode Mutual Fund Performance? The Role of Liquidity and Organization. *American Economic Review 94*(5), 1276–1302.

Chernozhukov, V. and H. Hong (2003). An MCMC approach to classical estimation. *Journal of Econometrics 115*(2), 293–346.

Chevalier, J. and G. Ellison (1997). Risk taking by mutual funds as a response to incentives. *Journal of Political Economy 105*(6), 1167–1200.

Choi, J. J., D. Laibson, and B. C. Madrian (2010). Why Does the Law of One Price Fail? An Experiment on Index Mutual Funds. *Review of Financial Studies 23*(4), 1405–1432.

Cremers, K. J. M. and A. Petajisto (2009). How Active Is Your Fund Manager? A New Measure That Predicts Performance. *Review of Financial Studies 22*(9), 3329–3365.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam (1998). Investor psychology and security market under and overreactions. *The Journal of Finance LIII*(6).

Fama, E. and K. French (2010). Luck versus Skill in the Cross-Section of Mutual Fund Returns. *The Journal of Finance LXV*(5).

Fama, E. and J. MacBeth (1973). Risk, return, and equilibrium: Empirical tests. *The Journal of Political Economy 81*(3), 607–636.

Ferson, W. and J. Lin (2011). Alpha and performance measurement: The effect of investor heterogeneity. *NBER working paper*.

Frazzini, A. and O. a. Lamont (2008). Dumb money: Mutual fund flows and the cross-section of stock returns. *Journal of Financial Economics 88*(2), 299–322.

French, K. (2008). Presidential address: The cost of active investing. *The Journal of Finance LXIII*(4).

Glode, V. (2011). Why mutual funds "underperform". *Journal of Financial Economics 99*(3), 546–559.

Greenwood, R. and S. Nagel (2009). Inexperienced investors and bubbles. *Journal of Financial Economics 93*(2), 239–258.

Haruvy, E., Y. Lahav, and C. N. Noussair (2007). Traders' Expectations in Asset Markets: Experimental Evidence. *The American Economic Review 97*(5), 1901–1920.

Hirshleifer, D. and J. Yu (2012). Asset pricing in production economies with extrapolative expectations. *SSRN* (March), 612–626.

Kacperczyk, M., C. Sialm, and L. Zheng (2005). On the industry concentration of actively managed equity mutual funds. *The Journal of Finance LX*(4).

Kacperczyk, M., C. Sialm, and L. Zheng (2006). Unobserved Actions of Mutual Funds. *Review of Financial Studies 21*(6), 2379–2416.

La Porta, R., J. Lakonishok, A. Shleifer, and R. Vishny (1997). Good news for value stocks: Further evidence on market efficiency. *Journal of finance LII*(2), 859–874.

Linnainmaa, J. T. (2013). Reverse Survivorship Bias. *The Journal of Finance 68*(3), 789–813.

Odean, T. (1998). Volume, volatility, price, and profit when all traders are above average. *The Journal of Finance LIII*(6).

Pastor, L. and R. Stambaugh (2012). On the size of the active management industry. *Journal of Political Economy* (August 2012).

Pástor, Ä., R. F. Stambaugh, and L. A. Taylor (2014). Scale and Skill in Active Management. *SSRN Electronic Journal*.

Pollet, J. and M. Wilson (2008). How does size affect mutual fund behavior? *The Journal of Finance LXIII*(6), 2941–2969.

Russo, J. E. and P. J. H. Schoemaker (1992). Managing overconfidence. *SloanManagement Review 33*.

Scheinkman, J. and W. Xiong (2003). Overconfidence and speculative bubbles. *Journal of political Economy 111*(6), 1183–1220.

Sharpe, W. F. (1966). Mutual fund performance. *Journal of business*, 119–138.

Sirri, E. and P. Tufano (1998). Costly search and mutual fund flows. *The journal of finance LIII*(5).

Staël Von Holstein, C.-A. S. (1972). Probabilistic forecasting: An experiment related to the stock market. *Organizational Behavior and Human Performance 8*(1), 139–158.

Tversky, A. and D. Kahneman (1974). Judgment under Uncertainty: Heuristics and Biases. *Science 185*(4157), 1124–1131.

Appendix

Appendix 1. Filtering Problem

In the appendix, the filtering problem of manager's ability from return history and non-return signal will be discussed.

I assume that manager's ability follows an AR(1) process.

$$m_{t+1} = m_t + \rho\left(\bar{m} - m_t\right) + \sigma_m \varepsilon_{t+1}^m,\ \varepsilon_{t+1}^m \sim \mathrm{N}\left(0,1\right)$$

The active return R_{t+1} is manager's ability plus noise term $\sigma_e \varepsilon_{t+1}^e$.

$$R_{t+1} = m_{t+1} + \sigma_e \varepsilon_{t+1}^e,\ \varepsilon_{t+1}^e \sim \mathrm{N}\left(0,1\right)$$

Excess fund return r_{t+1} is the net return of active return, cost, fee and noise term.

$$r_{t+1} = \frac{\mathrm{TP}_{t+1}}{S_t} = a\left(S_t\right) R_{t+1} - \frac{C\left(S_t a\left(S_t\right)\right)}{S_t} - f + \sigma_r \varepsilon_{t+1}^r,\ \varepsilon_{t+1}^r \sim \mathrm{N}\left(0,1\right)$$

Non-return signal s_{t+1} is informative on the manager's ability shock. The precision of the signal is θ.

$$s_{t+1} = \eta \varepsilon_{t+1}^m + \sqrt{1-\eta^2}\varepsilon_{t+1}^s,\ \varepsilon_{t+1}^s \sim \mathrm{N}\left(0,1\right)$$

$$\theta = \frac{\eta}{\eta + \sqrt{1-\eta^2}}$$

There are two signals: one is a signal from fund return and the other is a non-return signal. Following a discrete Kalman filter method (Liptser and Shiryaev, 1977), we can find the estimated manager's ability process. Kalman filter is basically orthogonal projection toward linear factors of signals. The observable signals of the manager's ability is as follows:

$$R_{t+1} = \left(1-\rho\right) m_t + \rho \bar{m} + \sigma_m \varepsilon_{t+1}^m + \sigma_e \varepsilon_{t+1}^e$$

$$s_{t+1} = \eta \varepsilon_{t+1}^m + \sqrt{1-\eta^2}\varepsilon_{t+1}^s$$

We apply the nonstationary Kalman filter model. The estimated process is

$$\hat{m}_{t+1} = (1-\rho)\,\hat{m}_t + \rho\bar{m} + \zeta_t\left(R_{t+1} - \left((1-\rho)\,\hat{m}_t + \rho\bar{m}\right)\right) + \psi_t s_{t+1}$$

Where

$$\zeta_t = \frac{(1-\rho)^2 P_t + \sigma_m^2 - \eta^2\sigma_m^2}{\left((1-\rho)^2 P_t + \sigma_m^2 + \sigma_e^2\right) - \eta^2\sigma_m^2},\ \psi_t = \frac{\eta\sigma_m\sigma_e^2}{\left((1-\rho)^2 P_t + \sigma_m^2 + \sigma_e^2\right) - \eta^2\sigma_m^2}$$

The error variance process is

$$P_{t+1} = (1-\rho)^2 P_t + \sigma_m^2 - \left(\zeta_t\left((1-\rho)^2 P_t + \sigma_m^2\right) + \psi_t\eta\sigma_m\right).$$

Appendix 2. Estimation Bias

In this section, the estimation bias of overconfidence and overextrapolation model is studied. Only the stationary case will be considered in this section. The nonstationary case is similar but do not have analytic solution. The true managerial ability process is the mean-reverting process. The agents estimate how much the ability depart from the mean. We can take out the mean from the inferring process without loss of generality. The manager's ability process follows:

$$m_{t+1} = (1-\rho)\,m_t + \sigma_m\varepsilon_{t+1}^m,\ \varepsilon_{t+1}^m \sim \mathrm{N}\,(0,1)$$

The active fund return is $R_{t+1} = m_{t+1} + \sigma_e\varepsilon_{t+1}^e = (1-\rho)\,m_t + \sigma_m\varepsilon_{t+1}^m + \sigma_e\varepsilon_{t+1}^e$. The estimated process of the overextrapolative(ρ_b) and the overconfident (η_b) agent is from the stationary Kalman filter model.

$$\hat{m}_{t+1} = (1-\rho)\,\hat{m}_t + \zeta_b\left(R_{t+1} - \left((1-\rho)\,\hat{m}_t\right)\right) + \psi_b s_{t+1}$$

where

$$\zeta_b = \frac{(1-\rho_b)^2\gamma + \sigma_m^2 - \eta_b^2\sigma_m^2}{\left((1-\rho_b)^2\gamma + \sigma_m^2 + \sigma_e^2\right) - \eta_b^2\sigma_m^2},\ \psi_b = \frac{\eta_b\sigma_m\sigma_e^2}{\left((1-\rho_b)^2\gamma + \sigma_m^2 + \sigma_e^2\right) - \eta_b^2\sigma_m^2}$$

The stationary error variance is

$$\gamma = \frac{-\left(1-(1-\rho_b)^2\right)\sigma_e^2 + \left(1-\eta_b^2\right)\sigma_m^2 + \sqrt{\left(\left(1-(1-\rho_b)^2\right)\sigma_e^2 + \left(1-\eta_b^2\right)\sigma_m^2\right)^2 + 4\left(1-\rho_b\right)^2\left(1-\eta_b^2\right)\sigma_m^2\sigma_e^2}}{2\left(1-\rho_b\right)^2}.$$

I claim that the estimation bias is a linear function of true manager's ability: $\hat{m}_t = (1+k)\,m_t + \nu_t$, where $m_t \perp \nu_t$. To prove that the bias is in this form, I will assume the estimation bias as $\hat{m}_t = (1+k)\,m_t + \nu_t$ and confirm that $\hat{m}_{t+1} = (1+k)\,m_{t+1} + \nu_{t+1}$.

$$\begin{aligned}
\hat{m}_{t+1} &= (1-\rho_b)\,\hat{m}_t + \zeta_b\left(R_{t+1} - ((1-\rho_b)\,\hat{m}_t)\right) + \psi_b s_{t+1} \\
&= (1-\rho_b)(1+k)\,m_t + (1-\rho_b)\,\nu_t \\
&\quad + \zeta_b\left(m_{t+1} + \sigma_e\varepsilon_{t+1}^e - (1-\rho_b)(1+k)\,m_t - (1-\rho_b)\,\nu_t\right) + \psi_b\left(\eta\varepsilon_{t+1}^m + \sqrt{1-\eta^2}\varepsilon_{t+1}^s\right) \\
&= \left((1-\rho_b)(1+k) + \zeta_b\left((1-\rho) - (1-\rho_b)(1+k)\right)\right)m_t \\
&\quad + \left\{(1-\zeta_b)(1-\rho_b)\,\nu_t + \zeta_b\left(\sigma_m\varepsilon_{t+1}^m + \sigma_e\varepsilon_{t+1}^e\right) + \psi_b\left(\eta\varepsilon_{t+1}^m + \sqrt{1-\eta^2}\varepsilon_{t+1}^s\right)\right\} \\
&= \frac{(1-\rho_b)(1+k) + \zeta_b\left((1-\rho) - (1-\rho_b)(1+k)\right)}{1-\rho}m_{t+1} \\
&\quad + \left(\psi_b\eta - \frac{(1-\zeta_b)(1-\rho_b)(1+k)}{1-\rho}\sigma_m\right)\varepsilon_{t+1}^m + (1-\zeta_b)(1-\rho_b)\,\nu_t \\
&\quad + \left\{\zeta_b\left(\sigma_e\varepsilon_{t+1}^e\right) + \psi_b\sqrt{1-\eta^2}\varepsilon_{t+1}^s\right\} \\
&= \left\{\frac{(1-\rho_b)(1+k) + \zeta_b\left((1-\rho) - (1-\rho_b)(1+k)\right)}{1-\rho} + \left(\psi_b\eta - \frac{(1-\zeta_b)(1-\rho_b)(1+k)}{1-\rho}\sigma_m\right)\frac{\rho^2}{\sigma_m}\right\}m_{t+1} \\
&\quad + \left(\psi_b\eta - \frac{(1-\zeta_b)(1-\rho_b)(1+k)}{1-\rho}\sigma_m\right)\sqrt{1-\rho^2}\varepsilon_{t+1}^{m^\perp} \\
&\quad + (1-\zeta_b)(1-\rho_b)\,\nu_t + \left\{\zeta_b\left(\sigma_e\varepsilon_{t+1}^e\right) + \psi_b\sqrt{1-\eta^2}\varepsilon_{t+1}^s\right\} \\
&= (1+k)\,m_{t+1} + \nu_{t+1}
\end{aligned}$$

I apply the conditional expectation for normal distribution[42] to ε_{t+1}^m and decompose ε_{t+1}^m as a function of m_{t+1}, $\varepsilon_{t+1}^m = E_{m_{t+1}}\left(\varepsilon_{t+1}^m\right) + \varepsilon_{t+1}^{m^\perp}$

We can find the constant k from the equation line 5 to 6:

$$k = \frac{\zeta_b(1-\rho) + (1-\rho)\,\psi_b\eta\frac{\rho^2}{\sigma_m}}{\zeta_b(1-\rho_b) - (\rho-\rho_b) + (1-\zeta_b)(1-\rho_b)\,\rho^2} - 1$$

42

$$E(X|Y=y) = \mu_X + \Sigma_{XY}\Sigma_{YY}^{-1}(y-\mu_Y)$$

$$Var(X|Y=y) = \Sigma_{XX} - \Sigma_{XY}\Sigma_{YY}^{-1}\Sigma_{YX}$$

[43]

The magnitude and sign of estimation bias depend on ζ_b and ρ_b. The constant k is positive at the SMM parameter estimates.

The bias of estimator is defined as:

$$\begin{aligned} Bias &= E_{m_t}\left(\hat{m}_t\right) - m_t \\ &= \left(1+k\right) m_t - m_t \\ &= k m_t \end{aligned}$$

The estimation error variance is defined as follows:

$$\sigma_\nu^2 = \frac{\left(\psi_b \eta - \frac{(1-\zeta_b)(1-\rho_b)(1+k)}{1-\rho}\sigma_m\right)^2 \left(1-\rho^2\right) + \zeta_b^2 \sigma_e^2 + \psi_b^2 \left(1-\eta^2\right)}{1-\left(1-\zeta_b\right)^2 \left(1-\rho_b\right)^2}$$

[44]

Figure 5 shows the graph of estimation biases of no-bias, overconfidence-only, overextrapolation-only, and both-biases model. The x-axis is manager's true ability and y-axis is the expected estimated ability conditional on true ability. At the estimated parameter, k for both-bias model is large and significantly different from zero, but k for other models are close to zero.

[Figure 5 about here.]

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[43]

$$1+k = \frac{\left(1-\rho_b\right)\left(1+k\right) + \zeta_b\left(\left(1-\rho\right) - \left(1-\rho_b\right)\left(1+k\right)\right)}{1-\rho} + \left(\psi_b \eta - \frac{\left(1-\zeta_b\right)\left(1-\rho_b\right)\left(1+k\right)}{1-\rho}\sigma_m\right)\frac{\rho^2}{\sigma_m}$$

$$\left(\zeta_b\left(1-\rho_b\right) - \left(\rho-\rho_b\right) + \left(1-\zeta_b\right)\left(1-\rho_b\right)\rho^2\right)\left(1+k\right) = \zeta_b\left(1-\rho\right) + \left(1-\rho\right)\psi_b \eta \frac{\rho^2}{\sigma_m}$$

$$k = \frac{\zeta_b\left(1-\rho\right) + \left(1-\rho\right)\psi_b \eta \frac{\rho^2}{\sigma_m}}{\zeta_b\left(1-\rho_b\right) - \left(\rho-\rho_b\right) + \left(1-\zeta_b\right)\left(1-\rho_b\right)\rho^2} - 1$$

[44]

$$\begin{aligned} \sigma_\nu^2 &= \mathrm{VAR}\left(\nu_{t+1}\right) \\ &= \mathrm{VAR}\left(\left(\psi_b \eta - \frac{\left(1-\zeta_b\right)\left(1-\rho_b\right)\left(1+k\right)}{1-\rho}\sigma_m\right)\sqrt{1-\rho^2}\varepsilon_{t+1}^{m^\perp} + \left(1-\zeta_b\right)\left(1-\rho_b\right)\nu_t + \left[\zeta_b\left(\sigma_e \varepsilon_{t+1}^e\right) + \psi_b\sqrt{1-\eta^2}\varepsilon_{t+1}^s\right]\right) \\ &= \left(\psi_b \eta - \frac{\left(1-\zeta_b\right)\left(1-\rho_b\right)\left(1+k\right)}{1-\rho}\sigma_m\right)^2 \left(1-\rho^2\right) + \left(1-\zeta_b\right)^2\left(1-\rho_b\right)^2 \sigma_\nu^2 + \zeta_b^2 \sigma_e^2 + \psi_b^2\left(1-\eta^2\right) \end{aligned}$$

Appendix 3. Empirical tests of alternative stories

In this section, I investigate other competing stories whether they can reconcile empirical findings of mutual fund dynamics. The previous literature have mostly focused on justification of negative fund returns only. This section discusses plausibilities of alternative models in perspectives of joint mutual fund dynamics including flows, returns and other stylized facts as well. The mutual fund information comes from CRSP mutual fund data which spans over around 50 years, from 1964 to 2012. I include actively manged US equity mutual funds and exclude index funds and foreign equity mutual funds. I follow general exclusion rules which are common in mutual fund literature, such as excluding funds smaller than 5 million dollars. I apply the CPI to deflate fund sizes using year 2006 as a base year.

Here I consider three alternative stories, which may not be mutually exclusive, but commonly referred:

1. The mutual fund managers do not have any ability to generate excess returns. The cross-sectional differences of mutual fund returns are all the luck.

2. Mutual fund managers provide hedges conditional on risk factors, and thus investors tolerates negative unconditional returns.[45]

3. A subset of managers has an ability to generate excess returns.

All three models do not assume diseconomies of scale. I will show that well-known empirical findings are difficult to be fitted in all three stories. All models have no implication on return-flow relationships, and cross-sectional distributions.

i. No managerial ability

When we focus only on aggregate returns and on an inexistence of long-run performance persistence, the model seems plausible as shown in Fama and French (2010). But empirics suggest that there exist a small degree of persistence in fund return. Table 3 shows results of return persistence regression, i.e. regress a fund return on a lagged fund return.[46] I control for a momentum strategy of mutual funds by including a momentum factor for risk adjustments. In table 3 panel A, a lagged return has a coefficient of 0.104 and is significant at the 1% level, which implies that 10.4% of excessive returns continue in the following year. In table 4, I calculate probabilities of successively being in the top decile and bottom decile of

[45] Ferson and Lin (2011) argue that mutual funds provide hedges against individual heterogeneous risks. But there is no evidence that fund returns generated from tradings span different dimension other than stock return risk space.

[46] Risk-adjusted returns are Carhart's four-factor adjusted returns.

fund returns. It shows that probabilities are significantly higher than the 10% level. In table 4, considerable amounts of persistence exist for at least one-year horizon. This is difficult to be reconciled by all the luck story because all the luck story do not generate any time-series covariance structure of fund returns.

[Table 11 about here.]

Table 11 shows before-fee risk adjusted mean fund returns. Both value-weighted and equal-weighted returns are higher than zero, but the value weighted return is 9 basis point and is insignificant at the 10% level. The equal weighted mean return is significantly higher than zero at the 1% level. If a fund performance is determined by only luck, individual funds cannot make better trading results even before fees regardless of fund sizes.

ii. Hedging conditionally

I assume that investors of mutual funds have same marginal utilities as investors of individual stocks. At an aggregate level, mutual funds are just pass-through vehicles of stocks. There is no reason why investors are segmented for stocks and mutual funds. Glode (2011) argues that mutual funds provide hedges against higher risk-premium, where risk premium is time-varying. This explanation only makes sense when fund returns provide hedges against higher risk-premium at an aggregate level. I assume that return-based risk factors span underlying macro factors even though there are inconclusive debates over macro factors. I use market, size, book-to-market, and momentum factors and make dummy variables whether these factors are negative or not. In each regression, I assume one of these factors as a representative risk factor. I run a following regression:

$$
\begin{aligned}
r_{i,t}^{e} &= \beta_m \left(r_{m,t} - r_{f,t}\right) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{MOM} MOM_t + I_{f<0} \\
&\quad + \beta_m^{f<0} \left(r_{m,t} - r_{f,t}\right) I_{f<0} + \beta_{SMB}^{f<0} SMB_t \cdot I_{f<0} + \beta_{HML}^{f<0} HML_t \cdot I_{f<0} + \beta_{MOM}^{f<0} MOM_t \cdot I_{f<0}.
\end{aligned}
$$

[Table 12 about here.]

Table 12 shows results of an aggregate hedging regression. Panel A to D are the results of value-weighted fund returns and panel E to H are the results of equal-weighted fund returns. If funds provide hedges against higher risk-premium[47], then we expect that mutual funds generate positive alphas during risky periods, that is, the coefficients of dummies for negative factor returns are greater than zero. There is no evidence that these coefficients are significantly greater than zero for all different risk factors.

[47] In general, high risk-premium coincide with low returns of risk factors.

[Table 13 about here.]

I also run a market-timing regression to find hedging evidences in aggregated fund returns. I include $\max(r_{mkt}, 0)$, $\max(r_{SMB}, 0)$, $\max(r_{HML}, 0)$ in regression of value-weighted aggregate returns and equal-weighted aggregate returns. The dependent variable of table 13 panel A is a value-weighted fund return and the dependent variable of table 13 panel B is an equal-weighted fund return. If funds successfully time the market on average, then the coefficients of timing variables have to be significantly positive. However, table 13 shows that timing variables are not significantly different from zero and timing variable for a size factor is even significantly negative at the 10% level. Overall, there are not much evidence that mutual funds provide hedge against high risk-premium.

iii. Only a subset of managers have ability

[Table 14 about here.]

If a subset of managers has an ability, we should observe persistent underperformance of a particular group or persistent outperformance of a particular group. Table 14 panel A shows mean return differences between top return decile groups and bottom return decile groups. I show that return differences of a sorting year and of subsequent two years. Following to the story, the bottom return decile group consists of no-ability fund managers more than other decile groups, and subsequent returns should be worse than other groups in an absolute term. However, return differences decrease substantially in the following year and become almost zero at year 2. It is difficult to reconcile the result if only a subset of managers has ability.

[Table 15 about here.]

Table 1: Summaries of actively managed mutual funds

The table reports the distribution of mutual fund return, size, flow and disappearing probabilities. The sample includes open-end U.S. funds excluding index funds. The sample period is from 1967 to 2012 and variables are annualized. Panel A presents aggregate value weighted and equal weighted risk-adjusted return. The risk adjusted return is

$$r_{i,t}^{e}=r_{i,t}-r_{f,t}-\beta_{m,t}\left(r_{m,t}-r_{f,t}\right)-\beta_{SMB,t}SMB_t-\beta_{HML,t}HML_t-\beta_{MOM,t}MOM_t.$$

where $\beta_{m,t}$, $\beta_{SMB,t}$,$\beta_{HML,t}$,$\beta_{MOM,t}$ are the estimates of three-year rolling window regression using monthly return. Fund return is an after-fee return. Panel B reports standard deviation of risk-adjusted return, 25th, 50th and 75th percentile return. Panel C presents relative fund size distribution. Panel D reports 25th, 50th and 75th percentile percentage flows. Pecentage flow of fund i is defined as $flow_{i,t+1}=\frac{TNA_{i,t+1}-TNA_{i,t}\times(1+r_{i,t+1})}{TNA_{i,t}}$, where $TNA_{i,t}$ is fund i's total net assets at time t, and $r_{i,t}$ is the raw return of fund i in period t. Panel E shows disappearing probabilities of different age groups. To calculate the disappearing probabilities, I count the number of disappearing funds for each age groups, and divide it by total number of same group funds at every year.

Panel A

variable	Value weighted return	Equal weighted return
estimate	-0.00678**	-0.00176
t-stat	(-2.318)	(-0.564)

Panel B

variable	Std. Dev. of raw return	Std. Dev. of risk-adjusted return	25th percentile return	50th percentile return	75th percentile return
estimate	0.10787***	0.08959***	-0.04442***	-0.00528**	0.03469***
t-stat	(12.22133)	(13.13513)	(-14.76169)	(-2.30116)	(8.25279)

Panel C

variable	50th percentile size /25th percentile size	75th percentile size /50th percentile size
estimate	3.15556***	3.33312***
t-stat	(16.58954)	(23.93762)

Panel D

variable	25th percentile flow	50th percentile flow	75th percentile flow
estimate	-0.12217***	-0.0218	0.19934***
t-stat	(-8.75322)	(-0.95619)	(3.72051)

Panel E

variable	disappearing prob for age≤2	disappearing prob for 3≤age≤6	disappearing prob for 7≤age≤10	disappearing prob for age≥11
estimate	0.01869*	0.02101*	0.01722*	0.01255**
t-stat	(1.96731)	(1.98067)	(1.87136)	(2.31639)

Table 2: Mutual fund return-flow regression

The table reports OLS coefficient estimates using the percentage flow as the dependent variable, which is defined as $flow_{i,t+1} = \frac{TNA_{i,t+1} - TNA_{i,t} \times (1 + r_{i,t+1})}{TNA_{i,t}}$, where $TNA_{i,t}$ is fund i's total net assets at time t, and $r_{i,t}$ is the raw return of fund i in period t. The percentage flow variable is winsorized at 5%. These regressions are run year by year, and standard errors and t-statistics are calculated from the vector of annual results, as in Fama and MacBeth (1973). I apply Newey-West covariance adjustment to vector of moments with 3-year lags. t-stats are given in parentheses below the coefficient estimates. The independent variables include risk-adjusted mutual fund return, return square, standard deviation of return, the log of fund i's total net assets, log fund TNA square, expense ratio, net objective flow, fund age, and interaction term between age and return. A fund risk-adjusted return is Carhart four-factor alpha from 3-year rolling regression of monthly returns. Total net assets are deflated by CPI using the base year of 2006. Net objective flow is an aggregated percentage flow of all the funds with the same investment objective. All the independent variables are 1-year lagged to fund flow except net objective flow. Panel B includes size quintile dummy variables and interaction term with return to test the heterogenous response to fund return by fund size. The sample includes open-end U.S. funds excluding index funds. The sample period is from 1967 to 2012 and variables are annualized.

variable	(A)	(B)
Intercept	0.58451***	0.55271***
	(3.56167)	(3.00037)
ret	1.63806***	1.73535***
	(4.23919)	(4.31744)
ret square	1.58906*	1.71502*
	(1.81715)	(1.7812)
std. dev. of ret	-0.53193	-0.37814
	(-0.7189)	(-0.47019)
log fund size	-0.13792***	-0.1277**
	(-3.4102)	(-2.53659)
log fund size square	0.00974***	0.00911**
	(3.09855)	(2.29978)
expense ratio	1.14923	0.94569
	(0.51583)	(0.41997)
net objective flow	0.39793***	0.39828***
	(4.36125)	(4.31581)
age	-0.0039***	-0.00381***
	(-5.74844)	(-5.75211)
age × ret	-0.02744***	-0.02439***
	(-4.54597)	(-4.12984)
dummy of size 2nd to 4th quintile		-0.01307
		-0.01028
dummy of size 5th quintile		(-0.40194)
		(-1.08377)
ret × dummy size 2nd to 4th quintile		-0.41853*
		(-2.17186)
ret × dummy size 5th quintile		-0.41853***
		(-3.09895)

Table 3: Mutual fund return predictability

The table reports OLS coefficient estimates using risk-adjusted mutual fund return as the dependent variable. These regressions are run year by year, and standard errors and t-statistics are calculated from the vector of annual results, as in Fama and MacBeth (1973). I apply Newey-West covariance adjustment to vector of moments with 3-year lags. t-stats are given in parentheses below the coefficient estimates. The independent variables include one year lagged risk-adjusted return, interaction term between return and return decile dummies, and interaction term between return and size quintile dummies. A fund risk-adjusted return is Carhart four-factor alpha from 3-year rolling regression of monthly returns. I use previous three year fund return to estimate the beta of risk factors and compute the risk-adjusted return using these beta. The sample includes open-end U.S. funds excluding index funds. The sample period is from 1967 to 2012 and variables are annualized.

variable	(A)	(B)	(C)
Intercept	-0.00668	-0.00786	-0.00631
	(-1.30026)	(-1.60824)	(-1.22845)
ret	0.10395*	0.13464**	0.12468***
	(2.00491)	(2.44233)	(3.7208)
ret × dummy ret bottom decile		-0.03764	
		(-0.01524)	
ret × dummy ret top decile		-0.31492	
		(-0.70505)	
ret × dummy size 2nd to 4th quintile			-0.02162
			(-0.04859)
ret × dummy size 5th quintile			-0.74813
			(-1.27679)

Table 4: Persistence of return difference

Panel A presents difference between median returns of top return quintile and bottom return quintile over different year horizon. Return difference at year 0 is the difference between median returns of top return quintile and bottom return quintile at return ranking year. Return difference at year 1 and year 2 are the median return difference of top return quintile and bottom return quintile for 1 year and 2 year post return ranking year. The fund return is a risk-adjusted return, i.e. Carhart four-factor alpha from 3-year rolling regression of monthly returns. Panel B reports probabilities of being in the same return decile for top return decile and bottom return decile funds. The benchmark probability for random mutual fund return case is 0.1. The sample includes open-end U.S. funds excluding index funds. The sample period is from 1967 to 2012 and variables are annualized.

Panel A

variable	return difference at year 0	return difference at year 1	return difference at year 2
estimate	0.17443***	0.04338***	0.01112
t-stat	(10.86137)	(6.30264)	(1.6726)

Panel B

Variable	prob top decile at year 1	prob bottom decile at year 1	prob top decile at year 2	probability bottom decile at year 2
estimate	0.21074***	0.20943***	0.16459***	0.19921***
t-stat	(11.51651)	(8.5324)	(8.69052)	(10.28953)

Table 5: Parameter estimates with both biases

The table summarizes the results of the estimation of overconfidence and overextrapolation model. The table reports estimates and standard errors of 11 model parameters. The standard errors are presented in the parantheses. The estimates are from SMM and simulation are repeated 10,000 times. In each simulation, 20,000 funds are generated and the simulation data length is 45 years same as the actual data length.

Parameter	Both Biases		No Biases		Overconfidence		Overextrapolation	
$\bar{m}$	0.22033	(0.07534)	0.0542	(0.02326)	0.33358	(0.08239)	0.33276	(0.04756)
σ_m	0.05687	(0.01594)	0.00894	(0.00251)	0.0586	(0.01352)	0.04166	(0.00959)
σ_e	0.17858	(0.03998)	0.09923	(0.02406)	0.29211	(0.03017)	0.25874	(0.03606)
θ	0.32321	(0.01532)	0.47067	(0.08731)	0.30024	(0.02501)	0.71657	(0.06009)
θ_b	0.75536	(0.05077)			0.73432	(0.05899)		
ρ	0.06733	(0.01381)	0.04293	(0.00863)	0.05884	(0.00564)	0.0796	(0.01697)
ρ_b	0.05189	(0.01448)					0.069	(0.01892)
x	1.88327	(0.17828)	1.42236	(0.06441)	1.47285	(0.06219)	1.4728	(0.10896)
σ_{m0}	0.19325	(0.02131)	0.03081	(0.0075)	0.14162	(0.03342)	0.077	(0.03446)
σ_r	0.02309	(0.00351)	0.00138	(0.00894)	0.0087	(0.00663)	0.00726	(0.00782)
a_{exit}	0.00191	(0.00372)	0.04364	(0.00517)	0.04264	(0.01328)	0.02388	(0.00724)

Table 6: Simulated moments with both biases

The table summarizes the results of the estimation of overconfidence and overextrapolation model. The table reports simulated values and actual values of the 15 estimation moments. The t-stat is reported in the last column and calculated using weighting matrix from the actual data. The estimates are from SMM and simulation are repeated 10,000 times. In each simulation, 20,000 funds are generated and the simulation data length is 45 years same as the actual data length. J-stat and p-value of the model is presented in the last two rows.

	Empirical Estimates		Both Biases		No Biases		Overconfidence		Overextrapolation	
	est.	std. err.	est.	t-stat	est.	t-stat	est.	t-stat	est.	t-stat
value-weighted mean return	-0.0067	(0.0029)	-0.00856	(-0.64343)	-0.00032	(2.18178)	-0.00169	(1.71197)	-0.00945	(-0.95001)
equal-weighted mean return	-0.0018	(0.0031)	-0.0051	(-1.00321)	-0.00023	(0.45758)	0.0013	(0.91911)	-0.00523	(-1.04209)
size 80th perc./size 50th perc.	4.4927	(0.0962)	4.53398	(0.24262)	4.55855	(0.38693)	4.1921	(-1.76478)	4.55151	(0.3456)
size 50th perc./size 20th perc.	4.2452	(0.1476)	4.43075	(0.69025)	4.31955	(0.27651)	4.85522	(2.26958)	4.62724	(1.42135)
flow 75th perc.-flow 25th perc.	0.3200	(0.0211)	0.32608	(0.17542)	0.22404	(-2.75592)	0.42255	(2.94652)	0.32807	(0.23248)
$\beta_{r}^{ret\,pred}$	0.1284	(0.0436)	0.13071	(0.06543)	0.00952	(-3.36557)	0.1373	(0.25191)	0.07559	(-1.49504)
$\beta_{r\times I_{bottom}}^{ret\,pred}$	-0.0331	(0.0403)	-0.05422	(-0.69934)	-0.00793	(0.83337)	-0.05951	(-0.87468)	-0.13496	(-3.37318)
$\beta_{r\times I_{top}}^{ret\,pred}$	-0.0070	(0.0624)	0.08025	(1.87352)	-0.00551	(0.03102)	0.02338	(0.65159)	0.03006	(0.79519)
$\beta_{r}^{ret-flow}$	1.3031	(0.1251)	1.66019	(1.86565)	1.40999	(0.55848)	1.64269	(1.77423)	1.65382	(1.83237)
$\beta_{r^2}^{ret-flow}$	1.7761	(0.5299)	3.0805	(2.16776)	3.56675	(2.97584)	3.31837	(2.56307)	2.66414	(1.47582)
$\beta_{age\times r}^{ret-flow}$	-0.0279	(0.0042)	-0.02992	(-0.40506)	-0.02578	(0.42838)	-0.03242	(-0.90647)	-0.02693	(0.19679)
Prob of top decile successively	0.2110	(0.0141)	0.19232	(-1.32938)	0.14283	(-4.85574)	0.22511	(1.00649)	0.18959	(-1.52434)
Prob of bottom decile successively	0.2092	(0.0201)	0.16887	(-2.14129)	0.2143	(0.26892)	0.22853	(1.02352)	0.14335	(-3.49492)
disappearing probability	0.0168	(0.0031)	0.01476	(-0.35243)	0.01836	(0.27266)	0.01175	(-0.87514)	0.0096	(-1.24806)
return prior to disappear	-0.04854	(0.017)	-0.03828	(1.11513)	-0.07384	(-2.74837)	-0.08008	(-3.42665)	-0.05119	(-0.28764)
χ^2			5.9963		25.046		21.5889		13.9228	
degree of freedom			4		6		5		5	
p-value			0.1994		0.0003		0.0006		0.0161	

Table 7: Parameter estimates of Berk and Green model

The table summarizes the results of the estimation of Berk and Green model. The table reports estimates and standard errors of 6 model parameters. The standard errors are presented in the parantheses. The estimates are from SMM and simulation are repeated 10,000 times. In each simulation, 20,000 funds are generated and the simulation data length is 45 years same as the actual data length.

Parameter	Berk and Green	
$\bar{m}$	0.23534	(0.02175)
$\sigma_{\bar{m}}$	0.0715	(0.00705)
σ_e	0.2779	(0.05152)
x	1.13028	(0.01274)
σ_r	0.0118	(0.01918)
a_{exit}	0.0386	(0.02148)

Table 8: Simulated moments of Berk and Green model

The table summarizes the results of the estimation of Berk and Green (2004) model. The table reports simulated values and actual values of the 15 estimation moments. The t-stat is reported in the last column and calculated using weighting matrix from the actual data. The estimates are from SMM and simulation are repeated 10,000 times. In each simulation, 20,000 funds are generated and the simulation data length is 45 years same as the actual data length. J-stat and p-value of the model is presented in the last two rows.

	Empirical Estimates		Berk and Green (2004)	
	est.	std. err.	est.	t-stat
value-weighted mean return	-0.0067	(0.0029)	-0.00081	(2.01574)
equal-weighted mean return	-0.0018	(0.0031)	0.00015	(0.573)
size 80th perc./size 50th perc.	4.4927	(0.0962)	4.39379	(-0.5805)
size 50th perc./size 20th perc.	4.2452	(0.1476)	4.94304	(2.59633)
flow 75th perc.-flow 25th perc.	0.3200	(0.0211)	0.22294	(-2.78724)
$\beta_{r}^{ret\,pred}$	0.1284	(0.0436)	-0.00148	(-3.6771)
$\beta_{r\times I_{bottom}}^{ret\,pred}$	-0.0331	(0.0403)	0.00278	(1.18827)
$\beta_{r\times I_{top}}^{ret\,pred}$	-0.0070	(0.0624)	-0.00083	(0.13157)
$\beta_{r}^{ret-flow}$	1.3031	(0.1251)	0.91843	(-2.00967)
$\beta_{r^2}^{ret-flow}$	1.7761	(0.5299)	1.03339	(-1.23428)
$\beta_{age\times r}^{ret-flow}$	-0.0279	(0.0042)	-0.03118	(-0.65675)
Prob of top decile successively	0.2110	(0.0141)	0.10802	(-7.33576)
Prob of bottom decile successively	0.2092	(0.0201)	0.13727	(-3.81766)
disappearing probability	0.0168	(0.0031)	0	(-2.91621)
return prior to disappear	-0.04854	(0.017)	0	(5.27401)
χ^2			62.0896	
degree of freedom			9	
p-value			0.0000	

Table 9: Return difference for flow quintiles

The table presents the fund returns of each flow quintile portfolios up to 1 year. The panel A is returns at year 0 and panel B is returns at year 1.

Panel A. Returns at year 0

		Actual data		Simulated data		Difference	
		estimate	t-stat	estimate	t-stat	estimate	t-stat
flow quintile	1	-0.01652**	(-2.25661)	-0.00725***	(-10.80)	-0.00927	(-1.261)
	2	-0.01537***	(-3.66727)	-0.00516***	(-17.08)	-0.01021**	(-2.430)
	3	-0.01114**	(-2.55381)	-0.00391***	(-13.55)	-0.00723	(-1.654)
	4	-0.00095	(-0.1862)	-0.00100***	(-4.85)	0.00005	(0.0098)
	5	0.02481***	(4.11291)	0.01740***	(13.77)	0.00741	(1.2023)

Panel B. Returns at year 1

		Actual data		Simulated data		Difference	
		estimate	t-stat	estimate	t-stat	estimate	t-stat
flow quintile	1	0.00248	(0.3256)	0.00673***	(9.73)	-0.00425	(-0.556)
	2	-0.01055**	(-2.52602)	-0.00227***	(-6.77)	-0.00828*	(-1.976)
	3	-0.00725*	(-1.68279)	-0.00317***	(-12.51)	-0.00408	(-0.945)
	4	-0.00705	(-1.5216)	-0.00335***	(-9.97)	-0.0037	(-0.796)
	5	-0.01436**	(-2.24444)	-0.00091*	(-1.43)	-0.01345**	(-2.091)

Table 10: Out of sample tests

Panel A presents return predictability regressions. The empirical estimates are the empirical results of Chen et al. (2004) and Cremers and Petajisto (2009). The simulated dataset of both biased model and no bias model are generated by the SMM parameter estimates. Each simulation includes 20,000 funds over 45 years. The estimates of the simulation are the result of ordinary least square of following equation:

$$\begin{aligned} r_{i,t+1} &= \beta \log\left(\text{size}_{i,t}\right) + \varepsilon_{i,t+1} \\ r_{i,t+1} &= \beta \text{active share}_{i,t} + \varepsilon_{i,t+1}. \end{aligned}$$

The dependent variables are risk-adjusted or benchmark-adjusted annualized fund return in decimal.

Panel B shows the difference between the counterfactual value-weighted return and the actual value-weighted return following Frazzini and Lamont (2008). Empirical estimates is for the case of mutual fund returns and 1-year flow in Frazzini and Lamont (2008). I replicate their calculation using risk-adjusted return and different sample periods over 1967-2012. Counterfactual value-weighted return is computed using the counterfactual fund size as a weight. The counterfactual fund size is the net of previous year counterfactual fund size and counterfactual fund flow.

$$\hat{\text{TNA}}_t^i = \left(1 + R_t^i\right) \hat{\text{TNA}}_{t-1}^i + \hat{F}_t^i$$

The counterfactual fund flows are defined as the reallocated actual fund flows proportional to the previous-year fund size.

$$\hat{F}_s = \frac{\text{TNA}_{t-1}^i}{\text{TNA}_{t-1}^{Agg}} F_s^{Agg}$$

The simulated data of both bias model is generated by the SMM parameter estimates and includes 20,000 funds over 45 years.

Panel A

	Chen et al. (2004)			Cremers and Petajisto (2009)		
	empirical estimates	simulation both biases	simulation no bias	empirical estimates	simulation both biases	simulation no bias
log size	-0.00336***	-0.00381***	0.00004			
	(3.02)	(-100.562)	(0.854)			
active share				0.0722***	0.06996***	-0.00198
				(2.53)	(97.804)	(-2.975)
dependent var.	benchmark adj. return			risk adj. return		

Panel B

	Frazzini and Lamont (2008)		
	empirical estimates	empirical replication	simulation both biases
Counterfactual	-0.0066***	-0.00108	-0.00097***
VW return	(-3.23)	(-0.062)	(-3.9279)

Table 11: Before-fee fund return

The table presents before-fee risk adjusted mean mutual fund return. The sample includes open-end U.S. funds excluding index funds. The sample period is from 1967 to 2012 and variables are annualized. The fund return is a risk-adjusted return, i.e. Carhart four-factor alpha from 3-year rolling regression of monthly returns. The value weighted risk-adjusted return is a lagged TNA weighted mean return and equal weighted risk-adjusted return is equally weighted mean return. Before-fee risk-adjusted fund return is calcultaed as net of after-fee risk-adjusted fund return and the level of total fees, i.e. expense ratio plus load.

variable	Value weighted return	Equal weighted return
estimate	0.0009	0.00867**
	(0.2866)	(2.60388)

Table 12: Aggregate hedging regression

The table reports the estimates of OLS regression. The dependent variables are monthly value-weighted fund return and equal-weighted fund return. To identify the time when risk premium is high, I generate a dummy variable whether a representative risk factor is negative or not. The independent variables include market, size, book-to-market, and momentum factors, dummy variable and interaction term between factors and dummy variable. In each regression, one of four factors is assumed as a representative risk factor. The regression model is

$$\begin{aligned} r_{i,t}^{e} &= \beta_m \left(r_{m,t} - r_{f,t}\right) + \beta_{SMB} SMB_t + \beta_{HML} HML_t + \beta_{MOM} MOM_t + I_{f<0} \\ &\quad + \beta_m^{f<0} \left(r_{m,t} - r_{f,t}\right) I_{f<0} + \beta_{SMB}^{f<0} SMB_t \cdot I_{f<0} + \beta_{HML}^{f<0} HML_t \cdot I_{f<0} + \beta_{MOM}^{f<0} MOM_t \cdot I_{f<0}. \end{aligned}$$

Panel A to D are the results of value weighted fund return and panel E to H are the results of equal weighted fund return. Panel A and E use market factor as a representative risk factor, panel B and F use size factor, panel C and G use book-to-market factor, and panel D and H use momentum factor. The sample includes open-end U.S. funds excluding index funds. The sample period is from 1967 to 2012. t-stats are given in parentheses below the coefficient estimates.

	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
dependent var	Value-weighted return				Equal-weighted return			
risk factor	mktrf	smb	hml	umd	mktrf	smb	hml	umd
Intercept	2.00E-05	0.00023	-0.00071	-0.00026	0.00102*	0.00179***	-0.00068	0.00061
	(0.04)	(0.51)	(-1.59)	(-0.62)	(1.66)	(3.26)	(-1.29)	(1.21)
dummy for factor<0	-0.00122	-0.00073	1.00E-05	-0.00032	-0.00229**	-0.00277***	0.0013	-0.00121
	(-1.65)	(-1.1)	(0.02)	(-0.52)	(-2.58)	(-3.49)	(1.64)	(-1.63)
mktrf	0.9055***	0.92505***	0.91847***	0.91967***	0.90998***	0.93282***	0.91768***	0.93551***
	(77.31)	(107.83)	(128.5)	(130.35)	(65.02)	(91.23)	(107.77)	(110.67)
mktrf × dummy for factor<0	0.00118	-0.01212	-0.00042	-0.00251	-0.0018	-0.00694	0.02695**	-0.01126
	(0.07)	(-1.12)	(-0.04)	(-0.24)	(-0.09)	(-0.54)	(2.11)	(-0.9)
smb	0.04905***	0.01672	0.04683***	0.06833***	0.16962***	0.1276***	0.18639***	0.20253***
	(5.23)	(1.27)	(4.77)	(6.96)	(15.14)	(8.12)	(15.93)	(17.22)
smb × dummy for factor<0	-0.00099	0.04938**	0.00353	-0.04303***	0.02615	0.04914*	-0.00877	-0.04901***
	(-0.06)	(2.33)	(0.23)	(-2.89)	(1.4)	(1.95)	(-0.49)	(-2.75)
hml	-0.02514**	-0.04073***	-0.03323**	-0.01328	-0.0076	-0.01708	0.00506	0.01978
	(-2.24)	(-3.52)	(-2.24)	(-1.23)	(-0.57)	(-1.24)	(0.29)	(1.53)
hml × dummy for factor<0	-0.02057	0.01026	-0.00532	-0.04866***	0.00665	0.02328	0.01764	-0.05276***
	(-1.26)	(0.64)	(-0.23)	(-3.04)	(0.34)	(1.22)	(0.63)	(-2.75)
umd	0.02548***	0.03144***	0.02851***	0.00811	0.02306***	0.02467***	0.02327***	0.00084
	(3.92)	(4.72)	(4.05)	(0.77)	(2.97)	(3.11)	(2.78)	(0.07)
umd × dummy for factor<0	-0.01135	-0.02406**	-0.01455	0.01367	-0.01314	-0.00977	-0.00748	0.01018
	(-1.02)	(-2.24)	(-1.39)	(0.98)	(-0.99)	(-0.76)	(-0.6)	(0.61)

Table 13: Market timing regression

The table reports the estimates of OLS regression. The dependent variables are monthly value-weighted fund return and equal-weighted fund return. The independent variables include market, size, book-to-market, and momentum factors, and timing variables, i.e. max(mktrf,0), max(smb,0), max(hml,0), and max(umd,0) . The sample includes open-end U.S. funds excluding index funds. The sample period is from 1967 to 2012.

	(A)	(B)
variable		
Intercept	-0.00023	0.000307
	(-0.55)	(0.61)
mktrf	0.91594***	0.9252***
	(94.87)	(79.81)
smb	0.07389***	0.21038***
	(4.9)	(11.62)
hml	-0.05049***	-0.01322
	(-3.11)	(-0.68)
umd	0.02899***	0.02491**
	(3.22)	(2.3)
max(mktrf,0)	0.00213	0.00724
	(0.13)	(0.36)
max(smb,0)	-0.04336*	-0.05063**
	(-1.88)	(-1.82)
max(hml,0)	0.02717	0.01674
	(1.09)	(0.56)
max(umd,0)	-0.01692	-0.01095
	(-1.07)	(-0.58)

Table 14: Return persistence

Panel A presents difference between mean returns of top return decile and bottom return decile over different year horizon. Return difference at year 0 is the difference between mean returns of top return decile and bottom return decile at return ranking year. Return difference at year 1 and year 2 are the mean return difference of top return decile and bottom return decile for 1 year and 2 year post return ranking year. The fund return is a risk-adjusted return, i.e. Carhart four-factor alpha from 3-year rolling regression of monthly returns. Panel B reports mean returns of top return decile and bottom return decile over different year horizon. Year 0 is the return ranking year, and year 1 and year 2 is 1 year and 2 year post return ranking year. The sample includes open-end U.S. funds excluding index funds. The sample period is from 1967 to 2012 and variables are annualized.

Panel A

Variable	return difference at year 0	return difference at year 1	return difference at year 2
estimate	0.28087***	0.05574***	0.01693
t-stat	(14.96916)	(5.13092)	(1.56928)

Panel B

Variable		year 0	year 1	year 2
top decile	estimate	0.14236***	0.02518***	-0.00033
return	t-stat	(11.32429)	(3.86546)	(-0.0582)
bottom decile	estimate	-0.13851***	-0.03056***	-0.01727**
return	t-stat	(-18.30765)	(-6.03162)	(-2.3339)

Table 15: Additional regressions: return predictability 1 year to 5 year

The table reports OLS coefficient estimates using risk-adjusted mutual fund return as the dependent variable. These regressions are run year by year, and standard errors and t-statistics are calculated from the vector of annual results, as in Fama and MacBeth (1973). I apply Newey-West covariance adjustment to vector of moments with 3-year lags. t-stats are given in parentheses below the coefficient estimates. The dependent variables are 1 year to 5 year forward one-year risk-adjusted return and independent variables include one-year risk-adjusted return, and interaction term between return and return decile dummies. A fund risk-adjusted return is Carhart four-factor alpha from 3-year rolling regression of monthly returns. I use previous three year fund return to estimate the beta of risk factors and compute the risk-adjusted return using these beta.

variable	(A) ret 1 year	(B) ret 1 year	(C) ret 2 year	(D) ret 2 year	(E) ret 3 year	(F) ret 3 year	(G) ret 4 year	(H) ret 4 year	(I) ret 5 year	(J) ret 5 year
Intercept	-0.00631	-0.00764*	-0.00764*	-0.00662*	-0.00604	-0.00684	-0.00507	-0.00732*	-0.00708	-0.00660
	(-1.46555)	(-1.92727)	(-1.80172)	(-1.89382)	(-1.47774)	(-1.70601)	(-1.15367)	(-1.74142)	(-1.53378)	(-1.56779)
ret	0.10618**	0.1284***	0.00417	0.01211	-0.03846	-0.05381	0.01849	-0.014	0.02419	0.01182
	(2.21192)	(2.91468)	(0.10723)	(0.25924)	(-0.81495)	(-0.85683)	(0.40546)	(-0.32954)	(0.65695)	(0.26247)
ret × dummy ret bottom decile		-0.033098		0.03269		0.028306		0.007963		0.037801
		(-0.81226)		(0.68392)		(0.51024)		(0.1462)		(0.76345)
ret × dummy ret top decile		-0.006954		-0.059134		0.033968		0.082419		0.020397
		(-0.11023)		(-0.97933)		(0.46103)		(1.68624)		(0.26965)

Figure 1: Fund size and returns

The figure presents the relation between fund sizes and fund returns. True managerial ability is assumed to be at the mean level (22.03%). The x-axis is the size of funds and the size of funds are corresponding to perceived abilities. Managers adjust their active shares according to their believes about managerial abilities and thus to fund sizes. The blue line shows the fund returns and the red line represent the active shares. Other parameters are set as the estimates of SMM in table 5.

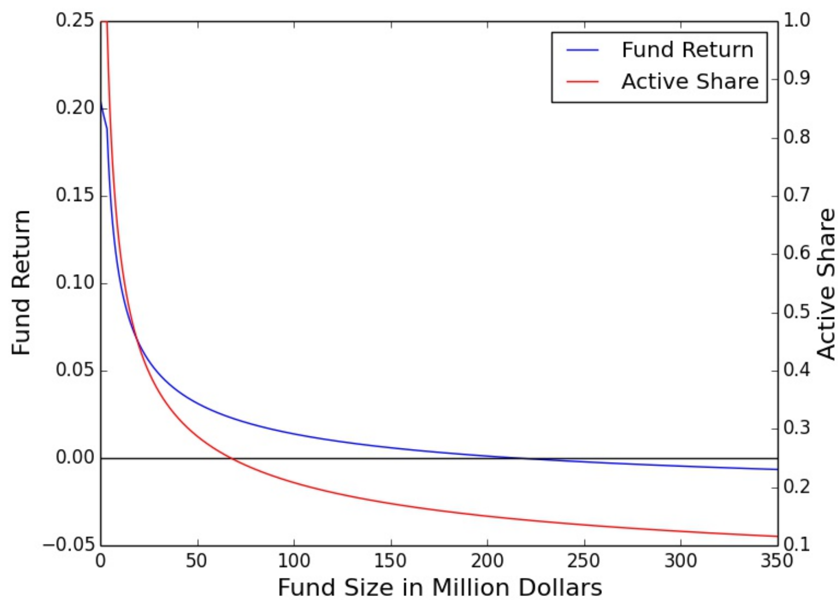
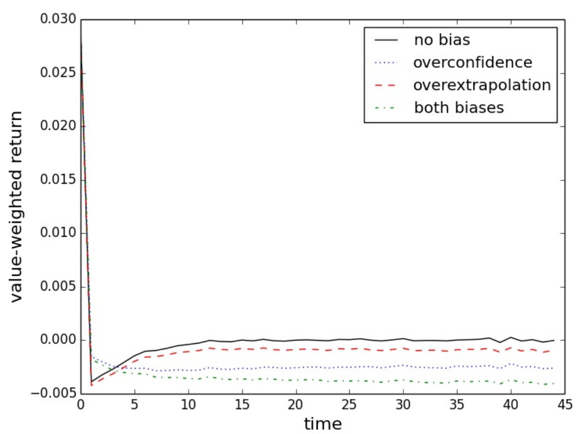


Figure 2: Impulse response of fund return for the positive active return error shock

The panel A and B present an impulse response of fund return to one standard deviation positive shock in active return error at time 0. After time 0, the active return errors are randomly drawn following the estimated distribution. Panel A displays the aggregate responses of no bias model, overconfidence only, overextrapolation only, and both bias model. Panel B presents the responses of bottom return quintile for the four models. The additional fund return error and initial uncertainty are suppressed to zero in all simulation. Other parameters are set as the estimates of SMM in table 5.

(a) Aggregate impulse response for active return error shock



(b) Bottom return quintile response for active return error shock

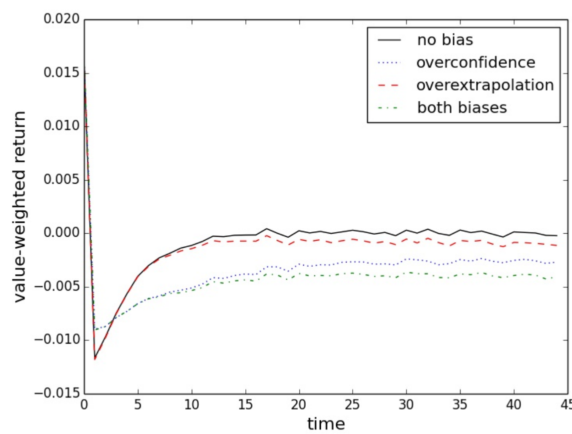


Figure 3: Simulated return flow relationship

The figure shows that return-flow relationship from the simulation. The fund flow at time t is the response to the fund return at time t-1. All the parameters are set as the estimates of SMM in table 5. Panel A presents time aggregate response to the return level from the piecewise linear regression. Panel B displays return flow relationship at different time period to show the time-varying sensitivity of flow response. The blue solid line is estimated at time 1 and the green dotted line is estimated at time 10.

(a) Aggregate return-flow relationship

(b) Time-varying return-flow relationship

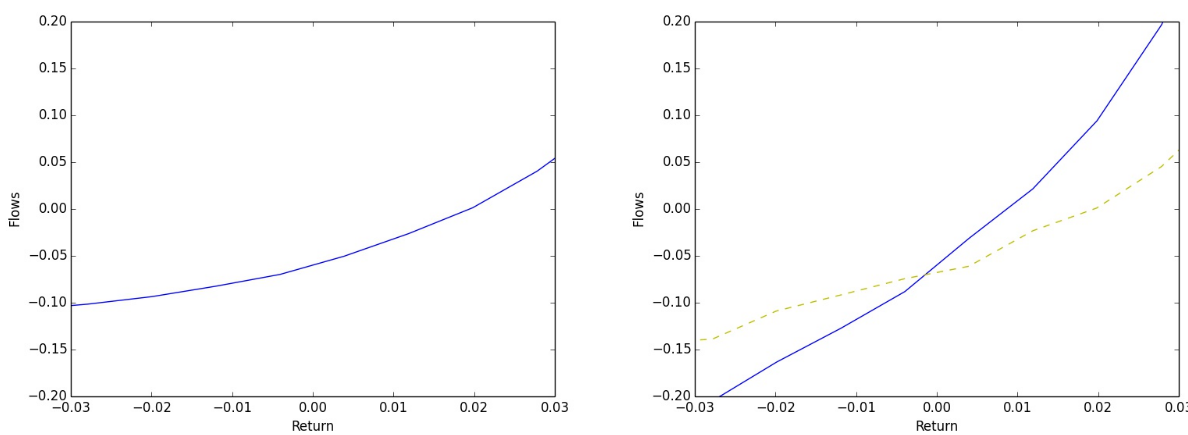
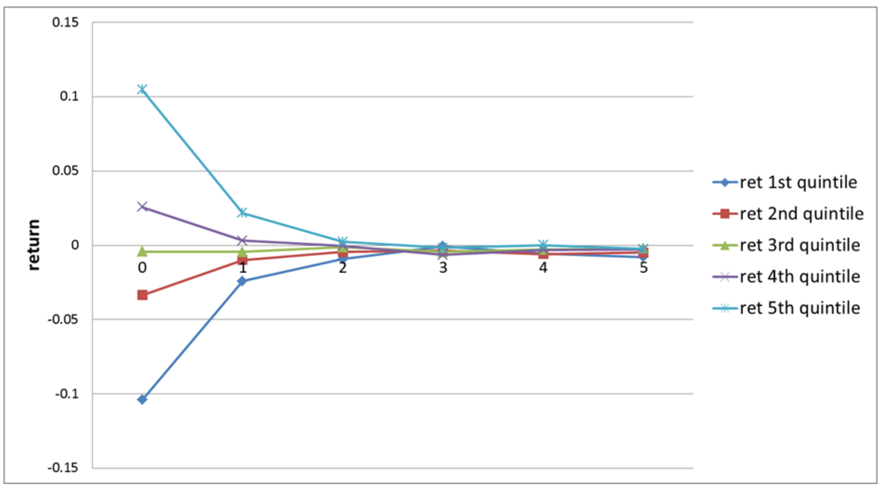


Figure 4: Return persistence graph for return quintiles

The figures show return time-series of time 0 return sorted groups. I rank the funds by fund returns at time 0 and track the fund returns of each return quintile portfolios. The figure tracks the returns up to year 5. A fund risk-adjusted return is Carhart four-factor alpha from 3-year rolling regression of monthly returns. I use previous three year fund return to estimate the beta of risk factors and compute the risk-adjusted return using these beta. The sub-figure A is from actual data and the sub-figure B is from simulation.

(a) Return persistence graph from actual data



(b) Return persistence graph from simulation

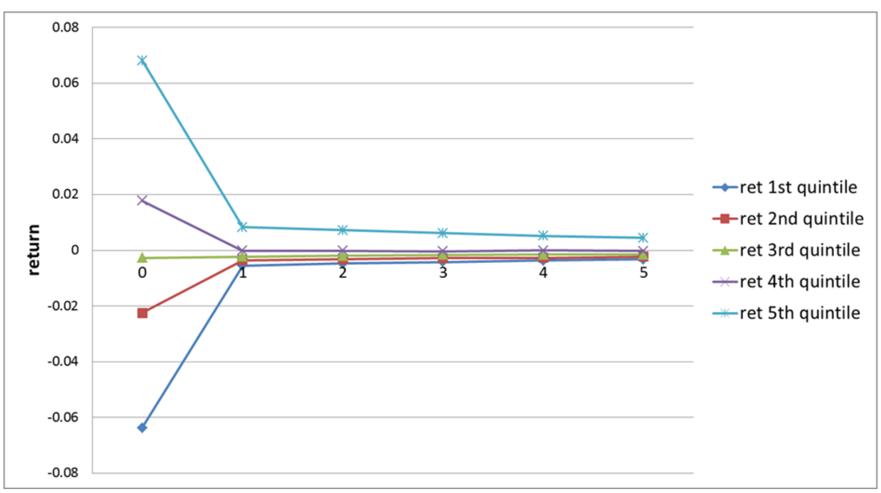


Figure 5: Estimation bias conditional on true ability

The figure presents the estimated ability conditional on true managerial ability. The lines represent a linear relationship between the true managerial ability and the estimated ability for no bias, overconfidence only, overextrapolation only, and both bias model. Parameters are set as the estimates of SMM in table 5.