

# Title: Liquidity, Bubbles and "Negative Bubbles": An Agency Model of Financial Intermediation\*

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## Abstract

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# 1 Introduction

This paper builds up on the work by Acharya and Naqvi (2012) who show that access to abundant liquidity exacerbates the risk-taking incentives of bank managers by encouraging them to give out excessive loans. Similar to Acharya and Naqvi (2012) for the purpose of our model we define liquidity as the total investment funds available to the financial intermediary. This is because in the model the investment funds are the main determinant of the cash holdings of the intermediary, which in turn are endogenously determined. Hence, instead of referring to the endogenous outcome (i.e. holdings of cash and cash equivalents) we refer to its driver (i.e. investment funds) as liquidity of the intermediary.

In this paper we build a theoretical model that generalizes the work done by Acharya and Naqvi (2012) by introducing heterogeneity on the asset side of the intermediary. More specifically, in our model the intermediary can invest in risky projects as well as safer projects. We are then able to show that when the intermediary is flush with liquidity the bank managers have an incentive to overinvest in the risky asset and concurrently underinvest in the safer asset. Moreover, we are able to show that the manager's investment preferences follow a certain pecking order: his first preference is to invest in risky assets (as they yield higher bonuses); his second preference is to hold cash or cash equivalents (so as to reduce the likelihood of liquidity shortfalls and hence audits); and finally his last preference is to invest in safer assets (since such assets yield lower or zero bonuses and at the same time are not as good a hedge against runs as cash or cash equivalents). It follows that in the presence of an agency problem the manager invests the minimum possible

amount in the safer (or medium risk) asset. Intuitively, overinvestment in the risky asset crowds out investment in the safer asset.

This result is consistent with the empirical findings of Chakraborty, Goldstein, and MacKinlay (2013). Using US data from 1988 through 2006 they find evidence that banks which increase their investment in the housing market simultaneously cut down on commercial lending. They find that an increase in housing prices is accompanied by a decrease in commercial lending. Coincidentally their sample period is also the period during which the intermediaries had abundant liquidity.

When considering the asset pricing implications of our model we are able to show that when the intermediary is flush with liquidity the overaggressive behavior of the manager is conducive to the formation of bubbles in the market for risky assets but concurrently a “negative asset price bubble” is formed in the market for safer assets. In other words, risky assets tend to be overpriced whilst safer assets are underpriced when intermediaries have access to abundant liquidity. We thus show that bubbles and negative bubbles can coexist in different markets due to the underlying agency problems in financial intermediaries.

As a case in point, before the advent of the 2008 financial crisis, when global banks had access to abundant liquidity due to loose monetary conditions there was a steep increase in the Spanish housing valuations due to the inflow of investments in its risky real estate sector. Fig. 1 depicts the steep increase in the price-to-rent ratio of Spanish houses. This increase in valuation relative to its historic long run mean was far greater than the increase in the price-to-rent ratio of U.S. real estate. On the other hand, the

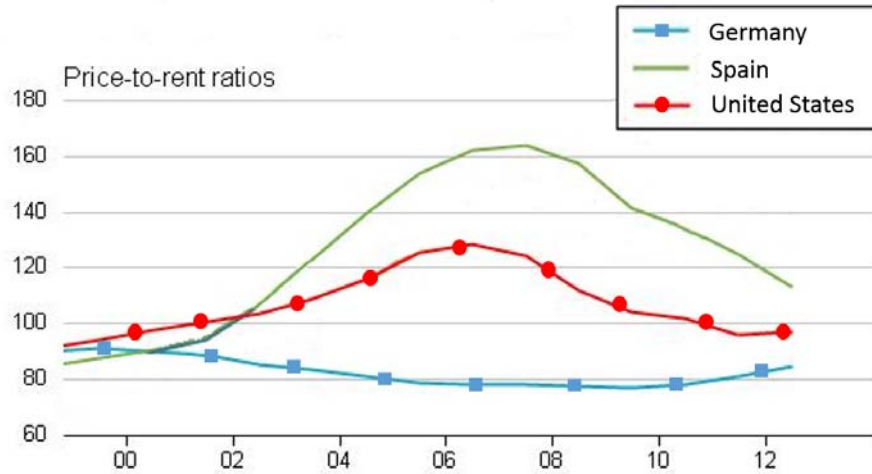


Figure 1: Comparison of house price valuations for Germany, Spain and U.S.  
Source: Thomson Reuters Datastream, OECD.

German real estate sector which is perceived to be safer experienced a dip in its valuation as measured by the price-to-rent ratio during this same period.

Finally, in this paper we also study how the leverage of a financial institution can potentially affect the risk-taking incentives of managers. We show that leverage can act as a disciplining device by curtailing the risk appetite of managers. We thus argue that regulators also need to consider that capital adequacy requirements by lowering leverage can potentially exacerbate the risk-taking incentives of managers.

The rest of the paper is organized as follows. The model is outlined in Section 2. In Subsection 2.1 we construct a base model of a financial intermediary in the presence of symmetric information. The intermediary receives investment funds from investors and then allocates these funds to projects

after setting aside some of the funds in the form of cash or cash equivalents. The intermediary can invest in risky projects as well as safer projects. The risky projects give higher returns in case of success but are characterized by a higher default risk as well as higher liquidity risk. More precisely, the risky projects have a lower probability of success and the cost of prematurely liquidating these projects is higher as compared to the safer projects. The intermediary suffers from early withdrawals whereby a fraction of investors withdraw their funds earlier in an interim period. If the cash holdings of the intermediary are insufficient to cater for the liquidity needs of the investors who withdraw early then the intermediary is forced to prematurely liquidate its projects. It prefers to liquidate the safer projects first given that these projects have a lower cost of premature liquidation.

In Subsection 2.2 we introduce asymmetric information between the manager of the intermediary and the principal. The manager needs to exert higher effort when investing in risky projects vis-a-vis safer projects. This is because risky projects entail higher (ex ante) screening costs as well as higher (ex post) monitoring costs. Since such effort is unobservable we show that the manager needs to be given higher bonuses for investing in risky assets relative to safer assets. However, such a contract encourages the manager to act overaggressively by overinvesting in risky assets and underinvesting in safer assets. To mitigate such behavior we allow for an audit which is conducted by the principal ex post to verify whether or not the manager had acted over aggressively. We show that since such audits are costly the principal will conduct an audit if and only if the liquidity shortfall suffered by the intermediary is sufficiently high. Intuitively, a high enough liquidity

shortfall sends a signal to the principal that the manager is more likely to have acted overaggressively.

Thus the manager of the intermediary faces a tradeoff: if he acts overaggressively he can potentially earn higher bonuses but in the event of a high enough liquidity shortfall the manager will be penalized. We then show that the manager will act overaggressively by overinvesting in the risky asset and underinvesting in the safer asset if and only if the liquidity (or the available investment funds) of the intermediary is sufficiently high. Intuitively, if the intermediary is awash with liquidity then the manager realizes that the likelihood of the intermediary suffering a liquidity shortfall in the interim period is significantly low. Consequently there is a high probability that the manager would be able to evade any penalties and earn high bonuses if the intermediary has access to abundant liquidity.

We then show that if the manager acts overaggressively his first preference would be to invest in risky assets; the second preference would be to invest in liquid assets like cash or cash equivalents; and finally he would invest the minimum possible amount in the safer or “medium risk assets”. Intuitively, by investing in risky assets the manager is able to earn high bonuses as long as no audit is conducted. Investment in liquid assets like cash and cash equivalents enables the manager to reduce the liquidity risk of his portfolio since such liquid assets are a good hedge against potential runs by investors. Thus retaining some investment funds in liquid assets effectively reduces the likelihood of an audit by reducing the probability of liquidity shortfalls. Finally, the manager will invest the minimum possible amount in the safer assets since such assets give low returns to the manager (in the form of lower

bonuses) and also the liquidity risk of such assets is higher than that of liquid assets.

In Section 3 we consider the asset pricing implications of our model. We define “fundamental” asset prices as those that arise in the absence of any agency frictions within the intermediary. An asset price “bubble” is said to exist when asset prices are above their fundamental values whilst an asset price “negative bubble” is said to exist when asset prices are below their fundamental values. We construct the optimal demand function of agents who borrow from the intermediary to invest in either risky or safer projects. Finally we solve for asset prices using the market clearing condition that the aggregate demand for assets should equal their supply. We then show that if the liquidity of the intermediary is sufficiently high then an asset price bubble is formed in the asset price of the risky asset and concurrently an asset price negative bubble is formed in the asset price of the safer asset. Intuitively, when the intermediary has access to abundant liquidity an agency problem is triggered which encourages the manager to overinvest in risky assets and underinvest in safer assets. Overinvestment in risky assets creates a bubble in the asset prices of risky assets whilst underinvestment in safer assets leads to a negative bubble being created in the market for safer assets. In other words, we show that bubbles and negative bubbles co exist in different markets due to agency problems within intermediaries.

In Section 4 we analyze how the leverage of an intermediary can affect the risk-taking incentives of managers. It is generally accepted that debt securities (such as demand deposit contracts) are more susceptible to runs as

compared to equity securities.<sup>1</sup> We show that if debt securities are more vulnerable to runs then the manager of an intermediary whose investments are financed largely by leverage is less likely to act overaggressively as compared to the manager of an intermediary where leverage is less significant. This apparently counterintuitive result can be explained by the disciplining nature of a debt contract. If an institution is financed largely by leverage then the manager realizes that the intermediary has little leeway to overinvest in risky assets and underinvest in safer assets due to the run-prone nature of securities like demand deposits. In the presence of significant leverage a liquidity shortfall is more likely to occur and this constrains the risk-taking ambitions of managers. This result is similar to Jensen (1986) whereby debt securities act as a disciplining device to constrain managers. Interestingly, in the light of this result, regulatory restrictions such as capital adequacy requirements may encourage more rather than less risk-taking on the part of managers.

Finally, Section 5 provides a discussion.

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<sup>1</sup>Cochrane (2014) argues that debt securities like demand deposits are run-prone contracts since debt is characterized by negative externalities whereby debt investors have an incentive to take out their investments before the other debtholders run. On the other hand, equity contracts are not run-prone contracts because equityholders cannot force the firm into bankruptcy for failure to pay immediately. Furthermore, equityholders realize that intermediaries have the right to delay payment, suspend convertibility, or pay in part, and thus in this case it is much harder for a run to develop.



## 2 The Model

### 2.1 The base model with symmetric information

We consider a model of a financial intermediary with three periods. At  $t = 0$ , risk-neutral investors invest an endowment of 1 unit each in the financial intermediary. There are a total of  $I$  investors and thus the intermediary receives  $I$  units of investment funds in the initial period. Each investor has a reservation utility of  $\bar{u}$ . Hence, the intermediary needs to ensure that the rate of return earned by investors or the promised yield,  $\rho_I$ , is such that investors earn an expected profit of at least  $\bar{u}$ .

After receiving investment funds, the intermediary makes investments in projects while setting aside a fraction of the funds received in the form of cash or cash equivalents,  $C$ . The intermediary can invest in two types of projects: “risky” projects or “safer” projects. Both projects either succeed or fail at  $t = 2$ . The intermediary is hit by a macroeconomic shock with probability  $1 - \theta$ , in which case all of the projects (including the safer projects) fail and the intermediary is insolvent. For simplicity, we assume that the intermediary is solvent and hence able to pay back the promised return to its investors (with probability  $\theta$ ) as long as it is not hit by the macroeconomic shock.

If the intermediary is solvent then the risky projects succeed with probability  $p$  but fail with probability  $1 - p$ . In case of failure, the risky projects yield a liquidation value of  $y$  as long as the intermediary is not hit by the macroeconomic shock. More precisely, the probability distribution of the

returns of the risky projects is given as follows:

$$\tilde{\rho}_R = \begin{cases} \rho_R & \text{with probability } \theta p \\ y & \text{with probability } \theta(1-p) \\ 0 & \text{with probability } 1-\theta \end{cases}, \quad (1)$$

where  $\rho_R$  is the (gross) rate of return from the risky projects charged by the intermediary. The probability distribution of the returns of the safer projects is given by

$$\tilde{\rho}_S = \begin{cases} \rho_S & \text{with probability } \theta \\ 0 & \text{with probability } 1-\theta \end{cases}, \quad (2)$$

where  $\rho_S$  is the (gross) rate of return from the safer projects charged by the intermediary. Since  $p < 1$  the safer projects have a higher probability of success.

After receiving the investment funds,  $I$ , the intermediary observes  $\theta$  and  $p$  and chooses the lending rates,  $\rho_i$  for  $i = R, S$ , which is the (gross) rate of return on the risky and safer projects. When setting the lending rates, the intermediary takes into account the demand function for loans, which is given by  $L(\rho_i)$ , where  $L'(\rho_i) < 0$ . The cash holdings of the intermediary are the residual after it makes all of its investments in the risky and safer projects:

$$C = I - L_R - L_S, \quad (3)$$

where for brevity  $L_R = L(\rho_R)$  is the loan demand for the risky asset and  $L_S = L(\rho_S)$  is the loan demand for safer assets.

Similar to Diamond and Dybvig (1983), a fraction of the investors, given by  $\tilde{x} \in [0, 1]$ , experience liquidity shocks and withdraw early at  $t = 1$ . The cumulative distribution function of  $\tilde{x}$  is given by  $F(x)$  while the probability

density function is given by  $f(x)$ . Each investor who withdraws early receives 1 unit of his endowment back at  $t = 1$ . Thus, the total withdrawals at  $t = 1$  are given by  $\tilde{x}I$ . If the total withdrawals exceed the amount of cash holdings,  $C$ , then the intermediary suffers a penalty cost which can be interpreted as a cost of premature liquidation of assets in order to service withdrawals. The penalty cost suffered by the intermediary in the event of a liquidity shortfall is given by:

$$\Psi = \begin{cases} \rho_S^p (xI - C) & \text{if } C < xI \leq C + L_S \\ \rho_S^p L_S + \rho_R^p (xI - C - L_S) & \text{if } xI > C + L_S \end{cases}, \quad (4)$$

where  $\rho_R^p > \rho_R > 1$ ,  $\rho_S^p > \rho_S > 1$ , and  $\rho_R^p > \rho_S^p > 1$ . The interpretation of the above formulation is as follows: when the total withdrawals,  $xI$ , are greater than the intermediary's cash holdings,  $C$ , but less than the sum of cash holdings and the amount invested in safer assets,  $C + L_S$ , then the intermediary does not need to liquidate the risky assets (which have a higher liquidation cost) and there will be partial or total liquidation of the safer assets in order to service withdrawals. The per unit cost of liquidating the safer asset is  $\rho_S^p$  and hence the penalty cost suffered by the intermediary will be  $\rho_S^p (xI - C)$ . However, if the total withdrawals,  $xI$ , exceed the sum of cash holdings and the amount invested in the safer assets,  $C + L_S$ , then the intermediary would need to completely liquidate the safer assets and it would also need to resort to partial or total liquidation of the risky assets, in order to meet the liquidity demands of its investors. The per unit liquidation cost of risky assets is given by  $\rho_R^p$  and thus, the total penalty cost suffered by the intermediary in this case would be given by  $\rho_S^p L_S + \rho_R^p (xI - C - L_S)$ .

In other words, the above formulation implies that the risky assets not

only have a higher default risk but also a higher liquidity risk since the cost of prematurely liquidating the risky assets is higher as compared to that of the safer assets. Hence if the intermediary suffers a liquidity shortfall then it initially prefers to cover the shortfall by liquidating the safer assets. However, if the number of withdrawals is large enough then the intermediary will eventually need to liquidate its risky assets. The implication is that the penalty cost of liquidity shortfalls increases with the amount of withdrawals.

We assume that as long as the intermediary is solvent it is able to repay the patient investors the promised rate of return at  $t = 2$ , whilst any residual returns are consumed by the principal (which could be interpreted as equityholders).

Let us suppose that the intermediary is run by a “money manager” who decides how to allocate the intermediary’s investment resources across assets. The manager needs to exert effort in order to sell loans and make investments. We assume that the effort cost of selling risky loans,  $e_R$ , is higher than the effort cost of selling safer loans,  $e_S$ , i.e.  $e_R > e_S$ . This is plausible because risky projects have a higher screening cost as well as a higher monitoring cost. Without loss of generality we normalize  $e_S = 0$ .<sup>2</sup> Since  $e_S = 0$ , we simplify our notation and write  $e_R = e$  thereby suppressing the subscript  $R$ . Henceforth,  $e$  refers to the effort cost of selling risky loans. We assume that the choice of effort is binary whereby  $e \in \{e^H, e^L\}$ . In other words, the manager can either exert high effort,  $e^H$ , or low effort,  $e^L$ , where  $e^H > e^L$ . We assume that it is in the interest of the principal to implement high effort.<sup>3</sup>

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<sup>2</sup>This simplifies the analysis. Nevertheless all our qualitative results are unchanged as long as  $e_R > e_S$ .

<sup>3</sup>The case where the principal wants to implement low effort is uninteresting because it

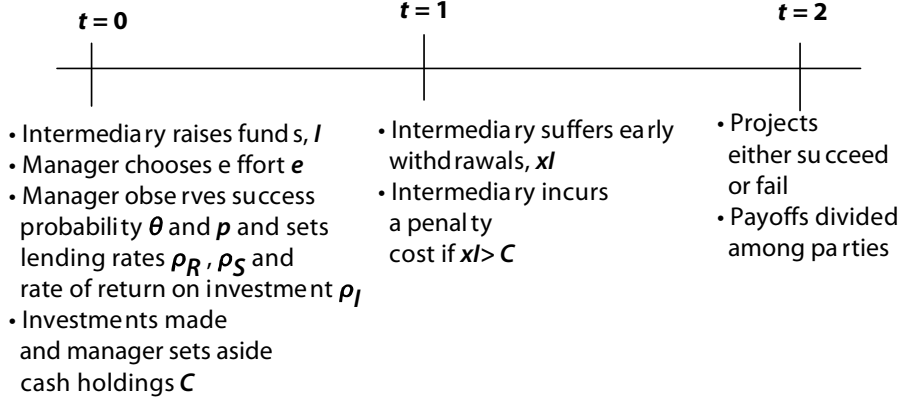


Figure 2: Base model: timeline of events

If the manager exerts high effort then he is able to sell more loans, i.e.  $[L(\rho_R) | e = e^H] > [L(\rho_R) | e = e^L]$ .<sup>4</sup>

The sequence of events is summarized in the time line depicted in Fig. 2.

Given this setup, in the case of symmetric information a money manager acting in the interest of the intermediary would solve the following problem:

$$\max_{\rho_R, \rho_S, \rho_I, C} \Pi = \pi - E[\Psi | e = e^H] \quad (5)$$

is simple to show that once we consider asymmetric information this can be implemented by simply offering a fixed wage to the manager. This is optimal only if the gains from the lower wage costs of inducing low effort outweigh the costs associated with lower profits. In practice, managers' wages are not fixed and they are often given an incentive to exert high effort. Henceforth, we only consider the interesting case where the principal finds in its interest to implement high effort.

<sup>4</sup>More specifically, if the manager exerts high effort, then for the same level of lending rate (and the same risk) he is able to sell more loans. Stated differently for the same price and quality a manager can sell more units if he exerts high effort. This implies that the demand function for risky loans shifts outwards when high effort is exerted.

subject to

$$E(\tilde{x}) + (1 - E(\tilde{x})) \left[ \theta \rho_I + (1 - \theta) \frac{E[\max(C - \tilde{x}I, 0) | e = e^H]}{(1 - E(\tilde{x}))I} \right] \geq \bar{u} \quad (6)$$

and

$$L(\rho_R) + L(\rho_S) + C = I, \quad (7)$$

where  $E(\cdot)$  is the expectations operator over  $\tilde{x}$  and  $\pi$  is given by

$$\begin{aligned} \pi = & \theta \{ \rho_S L(\rho_S) + p \rho_R [L(\rho_R) | e = e^H] + (1 - p) y \\ & - \rho_I I (1 - E(\tilde{x})) + E[\max(C - \tilde{x}I, 0) | e = e^H] \}. \end{aligned} \quad (8)$$

The above program says that a manager acting in the interest of the intermediary exerts high effort and chooses project lending rates, returns on investments and the level of cash holdings so as to maximize the expected profits of the intermediary,  $\pi$ , net of any penalty incurred in case of liquidity shortages and subject to participation constraint of the investors given by expression (6) and the budget constraint given by Eq. (7). An investor withdraws his funds early with a probability of  $E(\tilde{x})$  in which case he receives a payoff of 1. With a probability of  $(1 - E(\tilde{x}))$  the investor does not experience a liquidity shock in which case he receives a promised payment of  $\rho_I$  if the intermediary is solvent. In case of insolvency of the intermediary (which happens with probability  $1 - \theta$ ), any surplus cash holdings are divided amongst the patient investors. Thus expression (6) states that the investors must on average receive at least their reservation utility. Eq. (7) is a budget constraint of the intermediary which says that the total assets of the intermediary (i.e. sum of project loans and cash holdings) must equal the total investment funds. Eq. (8) represents the expected profit of the intermediary

exclusive of the penalty costs. With probability  $(1 - \theta)$  profits are zero since the intermediary is insolvent. With probability  $\theta$  the intermediary is solvent in which case the safe project gives the promised return of  $\rho_S L(\rho_S)$  while the risky project pays  $\rho_R L(\rho_R)$  (conditional on high effort) in case of success but yields the liquidation value  $y$  in case of failure. Thus in the case of solvency the intermediary's expected profit is given by the expected return from the projects minus the expected cost of investments ( $\rho_I I [1 - E(\tilde{x})]$ ) plus the expected value of net cash holdings at the end of the period (which is given by the last term of Eq. (8)).

We solve the above optimization problem and derive the first-best project lending rates, rate of return on investments, and level of cash holdings. The results are summarized in Proposition 1.

**Proposition 1** *The optimal gross lending rate for the safer project is given by*

$$\rho_S^* = \frac{\Pr[(\tilde{x}I \leq C) | e = e^H] + \rho_S^p \Pr[(\tilde{x}I > C^*) | e = e^H]}{\theta \left(1 - \frac{1}{\eta_S}\right)} \quad (9)$$

where  $\eta_S = -\rho_S L'(\rho_S) / L_S > 0$  is the elasticity of the demand for safer loans. The optimal gross lending rate for the risky project is given by

$$\begin{aligned} \rho_R^* = & \frac{\Pr[(\tilde{x}I \leq C) | e = e^H]}{\theta p \left(1 - \frac{1}{\eta_R}\right)} + \frac{\rho_S^p \Pr[(C < \tilde{x}I \leq C + L_S) | e = e^H]}{\theta p \left(1 - \frac{1}{\eta_R}\right)} \\ & + \frac{\rho_R^p \Pr[(\tilde{x}I > C + L_S) | e = e^H]}{\theta p \left(1 - \frac{1}{\eta_R}\right)} \end{aligned} \quad (10)$$

where  $\eta_R = -\rho_R [L'(\rho_R) | e = e^H] / [L_R | e = e^H] > 0$  is the elasticity of the demand for risky loans. The optimal gross rate of return on investments is

given by

$$\rho_I^* = \frac{(\bar{u} - E(\tilde{x}))I - (1 - \theta)E[\max(C^* - \tilde{x}I, 0) | e = e^H]}{\theta(1 - E(\tilde{x}))I}. \quad (11)$$

And, the optimal level of cash holdings is given by

$$C^* = I - L(\rho_S^*) - [L(\rho_R^*) | e = e^H]. \quad (12)$$

The lending rates in Proposition 1 are a (probability) weighted average of the per unit cost of liquidating the intermediary's assets scaled by default risk and adjusted for the elasticity of loan demand. As expected, the project lending rates increase as the per unit liquidation costs increases. Furthermore, the lending rates increase as the elasticity of loan demand decreases.

Taking the partial derivatives of the lending rates with respect to project risk and with respect to liquidity we get the following corollary to Proposition 1.

**Corollary 1** (*Risk effect*)  $\frac{\partial \rho_i^*}{\partial \theta} < 0$  for  $i = R, S$ , i.e., an increase in macro-economic risk  $(1 - \theta)$ , ceteris paribus, increases the equilibrium lending rate for both project types;  $\frac{\partial \rho_R^*}{\partial p} < 0$ , i.e., an increase in specific risk of the risky project  $(1 - p)$ , ceteris paribus, increases the equilibrium lending rate for the risky project. (*Liquidity effect*)  $\frac{\partial \rho_i^*}{\partial I} < 0$  for  $i = R, S$ , i.e., an increase in the liquidity of the intermediary, ceteris paribus, decreases the equilibrium lending rate for both project types.

The results of Corollary 1 are very intuitive. The first part of the corollary implies that the project loan rates are increasing in default risk. The last part of the corollary implies that as the intermediary's liquidity (defined by



its total investment funds) increases the expected penalty cost of liquidity shortage decreases and thus the intermediary passes some of this benefit to the borrowers in the form of a lower lending rate.

## 2.2 The model with asymmetric information

Now let us consider the case where there is asymmetric information between the principal and the manager such that the effort level of the manager is unobservable. We assume that, although the risky loans are affected by effort, they are not fully determined by it. This stochastic relation is necessary to ensure that effort level remains unobservable. More formally, we assume that the distribution of risky loan demand  $L(\rho_R)$  conditional on  $e^H$  first-order stochastically dominates the distribution conditional on  $e^L$ . In other words, for a given level of lending rate, the manager on average makes a higher volume of risky loans when he exerts high effort relative to the case in which he exerts low effort, i.e.  $E[L(\rho_R) | e_H] > E[L(\rho_R) | e_L]$ . As before, it is in the interest of the principal to implement high effort.

The manager earns an income,  $b$ , where  $b = b_R + b_S$ . The managerial income  $b$  can be interpreted as bonuses where  $b_R$  is the bonus earned from processing risky loans while  $b_S$  is the bonus earned from processing safer loans. The manager faces a penalty cost,  $\psi$ , if the principal conducts an audit and it is revealed that the manager had mispriced the loans by either setting the lending rates too high or too low relative to the case which maximizes the owner's expected profits. The managerial penalty is some proportion,  $\gamma$ , of the penalty cost incurred by the intermediary due to liquidity shortfalls. The manager has limited liability and thus the maximum penalty that can

be imposed on the manager is given by  $\bar{\psi}$ . It follows that the managerial penalty is given by  $\psi = \min(\bar{\psi}, \gamma\Psi)$ , where  $\gamma \in (0, 1]$ . Thus, the net wage earned by the manager is given by  $w = b^r + b^s - \psi$ .

Audits are costly and the cost of an audit is given by  $z$ . The probability that the principal will conduct an audit is given by  $\phi$ . The audit policy needs to be time-consistent. In other words, even though the principal would like to commit to a tough audit policy but because conducting audits is costly, it does so ex post only if it is desirable at that time.

The manager's utility function is represented by  $u(w, e)$ , where  $u_w(w, e) > 0$ ,  $u_{ww}(w, e) < 0$ , and  $u_e(w, e) < 0$  (where the subscripts denote the partial derivatives). This implies that the manager prefers more wealth to less, he is risk averse, and he dislikes high effort. More specifically we assume that the utility function is given by  $u(w, e) = v(w) - e$ , where  $v'(w) > 0$ ,  $v''(w) < 0$ . The manager's reservation utility is given by  $u^o$ .

The manager can observe the quality of the projects,  $\theta$  and  $p$ , as well as the specific level of investment funds available to the intermediary,  $I$ , at the time of setting the loan rate. However, this information is not available to the principal at the time of setting the contract. Hence, the principal cannot infer whether or not the manager had set the appropriate lending rates which maximize expected profits (unless the principal conducts an audit at  $t = 1$ ). We assume that the principal can observe the distribution of investment funds (instead of its exact level) which is given by  $J(I)$  and that the liquidity of an intermediary is non-verifiable ex post. This is plausible given that in practice managers have a lot of leeway regarding where to 'park' their funds. For instance, some of the liquidity can be lent out to other intermediaries

while at the same time the liquidity of other intermediaries can also make its way to the intermediary in question. Moreover, during the past two decades financial institutions have sharply expanded their off-balance sheet activities due to the pace of financial innovation. Such off-balance sheet items are particularly difficult to verify.<sup>5</sup> Examples of off-balance sheet liquidity include financing commitments, repurchase agreements, guarantees, foreign currency accruals and receivables, and exposure to special purpose vehicles amongst others.

The time line of events is summarized in Fig. 3. The chronology of events at  $t = 0$  is as follows. Principal offers contract to manager such that the high effort levels are chosen; manager chooses effort levels; manager receives deposits,  $I$ , and observes the riskiness of the projects,  $\theta$  and  $p$ ; and subsequently the manager sets the loan rates,  $\rho_S$  and  $\rho_R$ , as well as the rate of return on investments,  $\rho_I$ . At  $t = 0.5$ , for a given level of  $\rho_R$  the loan volume  $L(\rho_R)$  is realized, and cash holdings are set aside. At  $t = 1$  the intermediary could experience early withdrawals and in case of a liquidity shortfall the intermediary suffers a penalty cost. The principal then decides whether or not to conduct an audit. If an audit is conducted, the manager may or may not be penalized contingent on the outcome of the audit. Finally, at  $t = 2$ , the project payoffs are realized and divided amongst the parties given the contractual terms.

At the time of contracting, the manager has not yet received investment

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<sup>5</sup>Buljevich and Park (1999) report that by the end of 1991, the top ten U.S. commercial banks carried off-balance sheet related liabilities almost seven times that of their total combined assets.

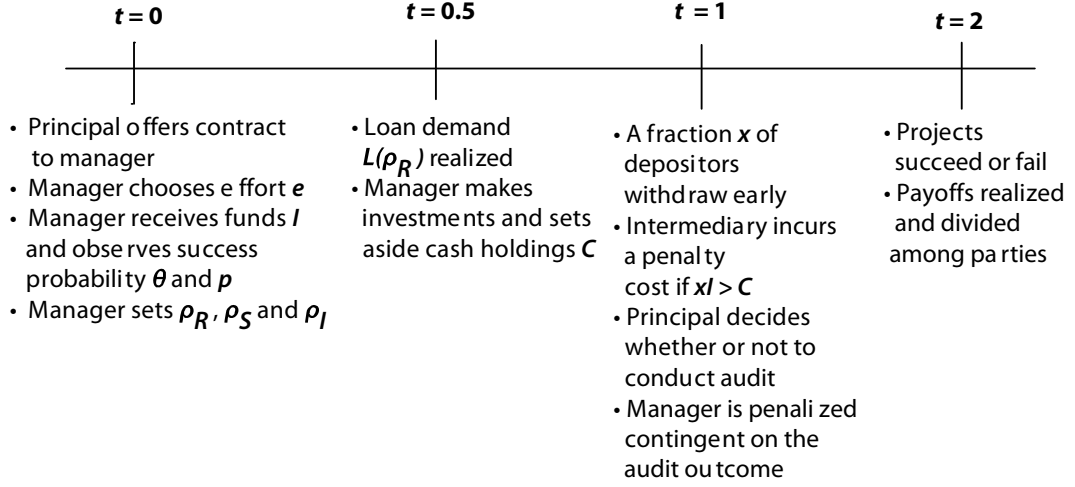


Figure 3: Timeline of events under asymmetric information.

funds and he sets the lending rate only after funds have been received and after observing projects' risk. This implies that when setting the lending rate, the manager takes into account the level of financial intermediary's liquidity,  $I$ , macroeconomic risk,  $\theta$ , and specific risk of the risky projects,  $p$ . However, this information is not available to the principal at the time of contracting and, hence the principal cannot enforce the optimal lending rates via an incentive compatible condition.

In this asymmetric information setting, the contract that the principal offers the manager specifies the compensation of the manager in the form of bonuses,  $b_i$  for  $i = R, S$ , penalties,  $\psi$ , as well as the 'audit policy',  $\phi$ . The audit policy is the likelihood with which the principal audits at  $t = 1$  conditional on the different scenarios. Because audit is costly, we consider time-consistent policies only. Moreover, when computing the optimal compensation scheme, the principal anticipates outcomes over different realiza-

tions of liquidity levels,  $I$ .

To determine the optimal managerial compensation scheme the principal needs to solve the following program:

$$\max_{b_R, b_S, \psi, \phi} \Pi - \bar{E}(b_R + b_S - \psi) - \bar{E}(z) \quad (13)$$

subject to

$$\bar{E}[v(b_R + b_S - \psi)] - e \geq u^o, \quad (14)$$

$$\bar{E}[u|e^H] > \bar{E}[u|e^L], \quad (15)$$

$$\psi \leq \min(\bar{\psi}, \gamma\Psi), \quad (16)$$

and

$$\phi \in [0, 1]. \quad (17)$$

where  $\bar{E}$  represents the expectations operator over the range of values of  $x$ ,  $L_R$ , and  $I$ .

The above program says that the principal chooses a compensation schedule so as to maximize his expected profits minus the expected compensation of the manager and minus the expected audit costs subject to a number of constraints. Constraint (14) is the participation constraint which says that the manager's expected utility must be at least equal to his reservation utility. Constraint (15) is the incentive compatibility constraint for inducing high managerial effort. Constraint (16) is the limited liability constraint and says that the managerial penalty cannot exceed  $\bar{\psi}$ . In fact by definition this constraint holds with equality.<sup>6</sup> Finally, constraint (17) imposes the condition that the probability of an audit lies between zero and one.

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<sup>6</sup>An upper limit on managerial penalty is plausible because if the penalty were extremely large it would not only violate limited liability of the manager but also an ex-

Let  $\ell = \max(xI - C, 0)$  represent the liquidity shortfall of the intermediary, if any. We can then prove the following proposition.

**Proposition 2** *The managerial compensation contract is such that bonuses for processing riskier loans,  $b_R$ , are increasing in the loan volume of risky loans,  $L_R$ . However, the bonuses for processing safer loans,  $b_S$ , are constant and thus do not vary with the loan volume of safer loans,  $L_S$ . Furthermore, the principal conducts an audit at  $t = 1$ , if and only if, the liquidity shortfall,  $\ell$ , suffered by the intermediary exceeds some threshold  $\ell^*$ . In other words, the optimal audit policy conditional on the realization of liquidity shortfall,  $\ell$ , is given by*

$$\phi|\ell = \begin{cases} 1 & \text{if } \ell > \ell^* \\ 0 & \text{otherwise} \end{cases} .^7 \quad (18)$$

The intuition is straightforward. Managerial bonuses are increasing in risky investments because the manager needs to be incentivized for exerting effort. On the other hand, since the manager does not need to exert effort to make safer investments he receives a fixed compensation for investing in safer assets irrespective of the loan volume of such assets.<sup>8</sup> By verifying whether or not the agent had acted over-aggressively when liquidity shortfalls tremendously large penalty would fail to satisfy the participation constraint of a risk-averse manager.

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<sup>7</sup>One can interpret  $\phi|\ell$  as the ex post audit probability, i.e., conditional on the realization of  $\ell$  the audit probability is equal to one if  $\ell > \ell^*$  and zero otherwise. Thus the ex ante audit probability at  $t = 0$  is given by  $\Pr(\ell > \ell^*)$ .

<sup>8</sup>In the case where the manager had to exert effort in order to make safer investments, his bonuses for investing in safer assets would also be increasing in the loan volume of safer loans. Nevertheless, his bonuses for making safer investments would be lower vis-a-vis his bonuses for investing in risky assets as long as risky investments required more effort on

are substantial ( $\ell > \ell^*$ ) and punishing him with the maximum penalty if it is inferred that he had misallocated resources, the principal discourages the agent from setting suboptimal loan rates. Importantly, if there are no liquidity shortfalls or liquidity shortfalls are minimal ( $\ell < \ell^*$ ), then that sends a signal to the principal that the manager was less likely to have acted overaggressively and to have reserved sufficient liquidity. Thus, in the absence of liquidity shortfalls the ‘return’ to the principal from incurring the cost of an audit is inadequate. This implies that there is no incentive ex post to conduct an audit unless liquidity shortfalls are sufficiently large.

The presence of a penalty upon audit creates a trade-off for the manager. The manager can increase his payoffs by making more risky investments. An increase in the volume of risky investments will crowd out the volume of safer investments. Since the manager gets a fixed wage from making safer investments he has an incentive to reduce the volume of safer investments but increase the volume of risky investments so as to increase his total compensation. However, an increase in the volume of risky investments can trigger a liquidity shortfall and subsequently the manager faces the risk of being audited and penalized.

### **2.2.1 Optimal loan rates under asymmetric information**

In the presence of asymmetric information, if the manager does not act over-aggressively and consequently acts in the interest of the principal, then he

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the part of the manager. This is likely to be the case given that making risky investments entail higher screening and monitoring costs.

solves the following problem for a given realization of  $I$ :

$$\max_{\rho_R, \rho_S, C, \rho_I} \pi - \hat{E} [\Psi | e = e^H] - \hat{E} [b + z | e = e^H], \quad (19)$$

subject to the participation constraint

$$\hat{E}(\tilde{x}) + (1 - \hat{E}(\tilde{x})) \left[ \theta \rho_I + (1 - \theta) \frac{\hat{E} [\max(C - \tilde{x}I, 0) | e = e^H]}{(1 - \hat{E}(\tilde{x})) I} \right] \geq \bar{u} \quad (20)$$

and the budget constraint

$$L(\rho_R) + L(\rho_S) + C = I, \quad (21)$$

where  $\hat{E}$  represents the expectation operator over the range of values of  $x$  and  $L_R$  and  $\pi$  is given by Eq. (8). In other words, a manager acting in the interest of the principal chooses loan rates, level of cash holdings, and rate of return on investments so as to maximize the gross profit of the intermediary net of the expected penalty costs associated with liquidity shortfalls, net of the expected wage and audit costs faced by the principal, and subject to the investors' participation constraint and the intermediary's budget constraint. If the manager is not acting overaggressively, he does not incur any penalty costs subsequent to an audit and, thus, the expected managerial penalty cost is zero conditional on the manager not acting over-aggressively.

**Proposition 3** *In the presence of asymmetric information, if the manager does not act overaggressively and, hence, there is no agency problem, then (for a given  $I$ ) the lending rates chosen by the manager are given by:*

$$\rho_S^{na} = \rho_S^* + \frac{\frac{\partial \hat{E}[b+z|e=e^H]}{\partial \rho_S}}{\theta L'(\rho_S)}, \quad (22)$$



and

$$\rho_R^{na} = \rho_R^* + \frac{\frac{\partial \hat{E}[b+z|e=e^H]}{\partial \rho_R}}{\theta \frac{\partial \hat{E}[L|e=e^H]}{\partial \rho_R}}, \quad (23)$$

where  $\rho_i^*$ , for  $i = R, S$ , are the first-best rates given by Eqs. (9) and (10). It follows that  $\rho_S^{na} > \rho_S^*$  and  $\rho_R^{na} > \rho_R^*$ .

The lending rates set by the manager in the presence of asymmetric information but in the absence of agency problems are higher than the first-best. The intuition is as follows. In the case of risky loans, an increase in the lending rate for risky loans lowers the loan volume of risky loans and thus reduces the associated bonuses which the principal has to pay to the manager given that managerial bonuses for risky loans are increasing in the loan volume of risky loans. Furthermore, audit costs are decreasing in the loan rate of both risky and safer loans. This is because an increase in lending rates reduces loan volume which in turn lowers the probability of liquidity shortfalls and thus decreases the expected audit costs. Consequently, a manager acting in the interest of the principal sets loan rates which are higher than the first-best. In short, in the presence of asymmetric information, the optimal loan rates that maximize the principal's expected profits are given by the second-best rates in Eqs. (22) and (23), which are both higher than the corresponding first-best rates.

### 2.2.2 Managerial agency problem

We will encounter a managerial agency problem if the manager maximizes his own expected utility instead of maximizing the principal's expected profits. In this case it can be shown that the manager will have a tendency to engage

in ‘overly aggressive behavior’. More specifically we define ‘overly aggressive behavior’ as follows.

**Definition 1** *A manager is said to engage in ‘overly aggressive behavior’ when he sets a lending rate such that  $\rho_R < \rho_R^{na}$  and  $\rho_S > \rho_S^{na}$ , where  $\rho_i^{na}$  is the optimal loan rate that maximizes the principal’s expected profits in the presence of asymmetric information. In other words, the manager engages in overly aggressive behavior when he ‘underprices’ the risky loan rate and ‘overprices’ the safer loan rate.*

The above definition implies that if a manager engages in overly aggressive behavior he will be overinvesting in risky assets but underinvesting in safer assets. In order to ascertain whether or not the manager will act over aggressively we solve for the manager’s optimization problem which is given by the following program:

$$\max_{\rho_R, \rho_S, C} E [v(b_R + b_S - \psi) | e = e^H] - e^H \quad (24)$$

subject to

$$L_R + L_S + C = I, \quad (25)$$

$$L_S \geq \underline{L}_S^j \quad \forall \theta^j, \quad (26)$$

where

$$\psi = \begin{cases} \min(\bar{\psi}, \gamma\Psi) & \text{if } \ell > \ell^* \text{ and } \rho_i \neq \rho_i^{na} \\ 0 & \text{otherwise} \end{cases}. \quad (27)$$

The above program says that the manager chooses his investment portfolio so as to maximize his expected utility conditional on high effort (24),

subject to the budget constraint (25). Condition (26) states that a minimum investment amount needs to be allocated to the safer asset for any given level of risk.<sup>9</sup> Condition (27) states that if the principal conducts an audit (which happens when  $\ell > \ell^*$ ), then the manager is imposed a penalty (which is a fraction  $\gamma$  of the intermediary's penalty cost  $\Psi$  but cannot exceed  $\bar{\psi}$  given limited liability) if it is inferred that the manager had not maximized the expected profits of the intermediary (which is the case when the manager sets loan rates which do not correspond to the rates that maximize the intermediary's expected profits under asymmetric information, i.e.  $\rho_i \neq \rho_i^{na}$ ).

After solving the above problem we can prove the following proposition.

**Proposition 4** *The manager will engage in overly aggressive behavior if and only if, the liquidity,  $I$ , of the intermediary is sufficiently high. Furthermore, if the manager acts over aggressively he will make the minimum possible investment in the safer asset and will overinvest in the risky asset.*

The proposition says that for high enough liquidity the manager has an incentive to overinvest in risky assets while underinvesting in safer assets. In other words, the agency problem is only actuated if the liquidity ( $I$ ) of

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<sup>9</sup>For example, given a risk level of  $1 - \theta^j$  the manager needs to invest at least  $\underline{L}_S^j$  in the safer asset, where  $\underline{L}_S^j$  is decreasing in risk. Such constraints exist in practice to satisfy internal risk management constraints as well as external regulatory requirements. Alternatively, we can simply replace this more general condition with a non-negativity constraint  $L_S \geq 0$  without affecting any of our results. This is effectively a short selling constraint and in the absence of such a constraint the manager will have an incentive to short sell the safer invest and reallocate the proceeds between the risky asset and cash holdings.

the intermediary is high enough. The intuition behind the above result is as follows. In the presence of excessive liquidity the probability that the intermediary will suffer a liquidity shortfall is very low and hence it is unlikely that the manager will be audited. A rational manager understands this and thus when he observes that the intermediary is flush with liquidity he has an incentive to overinvest in the risky assets so as to increase his bonuses. In other words, high liquidity is tantamount to insurance since it provides a buffer to the manager. In contrast, for low enough liquidity an audit is more likely and thus the manager refrains from acting over aggressively.

Due to the manager's limited liability, an upper bound exists on the penalty that can be imposed on the manager. Of course, in the absence of limited liability the principal could avoid an agency problem by imposing an arbitrarily large penalty if it was inferred that the manager had acted over-aggressively. However, limited liability on the part of the manager implies that such extreme punishments cannot plausibly be implemented and consequently, agency problems will arise for high enough levels of intermediary's liquidity.

The above proposition says that not only does the manager overinvest in the risky asset, but he also underinvests in the safer asset. Intuitively, overinvestment in the risky asset crowds out investment in the safer asset, which is conducive to underinvestment in the safer asset. It is interesting to note that the manager has no incentive whatsoever to invest in the safer asset. This is because he gets higher bonuses from investing the same amount in the risky asset while he gets lower or no bonuses from investment in the safer asset given that investments in safer assets entail lower screening and

monitoring costs. In fact, the manager is better off by retaining funds in the form of cash holdings rather than investing those funds in the safer asset. This is because, cash holdings provide a buffer against runs and lower the expected penalty cost that the manager will suffer. On the other hand, investments in the safer asset yield no bonuses and at the same time have a higher liquidation cost vis-a-vis cash. Thus the manager will only invest the minimum amount necessary in the safer asset.<sup>10</sup>

We then have the following corollary to Proposition 4.

**Corollary 2** *If the manager acts over aggressively he follows the following pecking order when making portfolio allocations: The first preference is to invest in risky assets; the second preference is to invest in the safest asset like cash or cash equivalents; and finally the least desirable investment allocation is in “medium risk assets” (which are safer than risky assets but are riskier than cash or cash equivalents).*

### 3 Bubbles and “negative bubbles”

Next we consider the asset pricing implications of our results. We define the fundamental asset price as the price that would prevail in the absence of any agency problems. A “bubble” would then arise if the actual asset price exceeds the fundamental price. Conversely, a “negative bubble” would be created if the actual asset price is lower than the fundamental price. To

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<sup>10</sup>In fact, in the absence of the short-sale constraint (26) the manager will short sell the safer asset as much as is possible and reallocate the proceeds between the risky asset and cash.

facilitate this comparison we model the asset demand by agents who borrow from financial intermediaries and subsequently invest the borrowed sum in risky or safer projects.

We assume that there exists a continuum of risk-neutral borrowers who have access to either risky or safer projects. These agents have no wealth and, hence, need to borrow from financial intermediaries to make investments in projects. We analyze the behavior of a representative borrower who has access to a project of risk type  $i$ , where  $i = R, S$  denotes that the project is either a risky project or a safer project. Analysis of a representative behavior implies that the equilibrium is symmetric and all borrowers of type  $i$  will choose the same portfolio. This also implies that the intermediary cannot discriminate between borrowers of the same type by conditioning the terms of the loan on the amount borrowed. Hence, borrowers can borrow as much as they like at the going rate of interest.

Asset  $i$  returns a cash flow (or cash flow equivalent of consumption) of  $X_i$  per unit with a probability of  $\rho_i$ , where as defined in Subsection 2.1, the success probability of the risky project is given by  $\omega_R = \theta p$  while the success probability of the safer project is given by  $\omega_S = \theta$ , where  $\omega_S > \omega_R$  since  $p < 1$ .<sup>11</sup> We make the usual assumption that the cash flow,  $X_i$ , is sufficiently high so that the borrower earns a positive payoff net of any investment costs contingent on the success of the project. Let  $P_i$  denote the price of one unit of the asset. Let  $Y_i^d$  denote the number of units of asset  $i$  demanded by the representative borrower and  $\tilde{Y}_i^s(P_i)$  denote the total supply of the

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<sup>11</sup>In case of failure, the risky project yields  $y$  but this return accrues to the intermediary owing to default and thus the borrower gets zero in the event of failure.

asset. The supply of asset  $i$ ,  $\tilde{Y}_i^s(P_i)$ , is stochastic, where  $\tilde{Y}_i^{s'}(P_i) > 0$  for any realization of  $Y_i^s(P_i)$ . In other words, if asset prices are high, then the supply of the asset increases. As in Acharya and Naqvi (2012) and Allen and Gale (2000), we assume the borrowers face a non-pecuniary cost of investing in projects  $t_i(Y_i^d)$ , which satisfies the usual neoclassical properties:  $t_i(0) = t'_i(0)$ ,  $t'_i(Y_i^d) > 0$ , and  $t''_i(Y_i^d) < 0$  for all  $Y_i^d > 0$ . This serves to restrict the size of the individual portfolios and ensures the concavity of the borrower's objective function. Alternatively, we can assume that the borrowers are risk averse which would lead to similar results.

The problem faced by the representative borrower is to choose the amount of borrowing so as to maximize his expected profits:

$$\max_{Y_i^d} \omega_i [X_i Y_i^d - \rho_i P_i Y_i^d] - t_i(Y_i^d) \quad (28)$$

subject to the market-clearing condition

$$Y_i^d = Y_i^s. \quad (29)$$

Expression (28) represents the expected profit of the representative borrower. In the event of success (with probability  $\omega_i$ ) the borrower receives a return of  $X_i Y_i^d$  on the units invested but needs to pay interest of  $\rho_i$  on his borrowings ( $P_i Y_i^d$ ) and also suffers the investment cost  $t_i(Y_i^d)$ . Thus the borrower chooses how much to invest in his project so as to maximize his expected profit given the market clearing condition that aggregate demand equals supply.

The first order condition of problem (28) is

$$\omega_i [X_i - \rho_i P_i] - t'_i(Y_i^d) = 0. \quad (30)$$

Solving for  $P_i$  we get

$$P_i = \frac{\omega_i X_i - t'_i(Y_i^d)}{\omega_i r_i}. \quad (31)$$

Finally, substituting  $Y_i^d = Y_i^s$  and letting  $\tau_i(Y_i^d) = t'_i(Y_i^d)$  denote the marginal investment cost, the equilibrium unit asset price is given by the following fixed-point condition

$$P_i^* = \frac{\omega_i X_i - \tau_i(Y_i^s(P_i^*))}{\omega_i \rho_i}. \quad (32)$$

The above expressions says that the equilibrium asset price is the discounted value of the expected cash flows net of the investment cost. Substituting  $i = R$  and  $\omega_R = \theta p$ , the equilibrium asset price of the risky asset is given by

$$P_R^* = \frac{\theta p X_R - \tau_R(Y_R^s(P_R^*))}{\theta p \rho_R}, \quad (33)$$

and substituting  $i = S$  and  $\omega_S = \theta$  we get the equilibrium asset price of the safer asset which is given by

$$P_S^* = \frac{\theta X_S - \tau_S(Y_S^s(P_S^*))}{\theta \rho_S}. \quad (34)$$

It can then be shown that there exists a one-to-one mapping from the lending rate,  $\rho_i$ , to the asset price,  $P_i$ . Taking the derivative of the equilibrium asset price with respect to the loan rate we get:

$$\frac{dP_i^*}{d\rho_i} = -\frac{X_i}{\rho_i^2} + \frac{\tau_i(Y_i^s(P_i^*))}{\omega_i r_i^2} - \frac{\tau'_i(Y_i^s(P_i^*)) Y_i^{s'}(P_i)}{\omega_i r_i} \frac{dP_i^*}{d\rho_i}. \quad (35)$$

Rearranging and simplifying Eq. (35) we get

$$\frac{dP_i^*}{d\rho_i} \left[ 1 + \frac{\tau'_i(Y_i^s(P_i^*)) Y_i^{s'}(P_i)}{\omega_i \rho_i} \right] = -\frac{P_i^*}{\rho_i}. \quad (36)$$

Since  $\tau'_i(\cdot) = t''_i(\cdot) > 0$ ,  $Y_i^{s'}(\cdot) > 0$ , and  $P_i^* \geq 0$ , it follows that  $\frac{dP_i^*}{d\rho_i} < 0$ . This implies that  $\frac{dY_i^s(P_i^*)}{d\rho_i} < 0$ . Thus in equilibrium given the market-clearing condition (i.e.  $Y_i^d = Y_i^s(P_i^*(\rho_i))$ ) the asset demand,  $Y_i^d$ , is decreasing in  $\rho_i$ .



Let  $\rho_i^{na}$  denote the fundamental (gross) lending rate which is the rate obtained in the absence of any agency problems, where  $\rho_S^{na}$  is given by Eq. (22) and  $\rho_R^{na}$  is given by Eq. (23). Then the fundamental asset price is given by the following fixed-point condition

$$\bar{P}_i^* = \frac{\omega_i X_i - \tau_i(Y_i^s(\bar{P}_i^*))}{\omega_i \rho_i^{na}}. \quad (37)$$

Thus the fundamental asset price of the safer project is given by

$$\bar{P}_S^* = \frac{\theta X_S - \tau_S(Y_S^s(\bar{P}_S^*))}{\theta \rho_S^{na}}, \quad (38)$$

whilst the fundamental asset price of the risky asset is given by

$$\bar{P}_R^* = \frac{\theta p X_R - \tau_R(Y_R^s(\bar{P}_R^*))}{\theta p \rho_R^{na}}. \quad (39)$$

Having derived fundamental asset prices we can now formally define bubbles and negative bubbles as follows:

**Definition 2** *An asset price bubble is formed whenever  $P_i^* > \bar{P}_i^*$ .*

**Definition 3** *An asset price “negative bubble” is formed whenever  $P_i^* < \bar{P}_i^*$ .*

Comparing the equilibrium asset price,  $P_i^*$ , given by Eq. (32) with the fundamental asset price,  $\bar{P}_i^*$ , given by Eq. (37), it can be noted that  $P_i^* > \bar{P}_i^*$  as long as  $\rho_i < \rho_i^{na}$ . Conversely,  $P_i^* < \bar{P}_i^*$  as long as  $\rho_i > \rho_i^{na}$ . In words, a lending rate lower than the fundamental rate creates a high demand for the asset, which leads to an increase in asset prices over and above the fundamental values. However, a lending rate higher than the fundamental

rate reduces the demand for the asset, which leads to asset prices being suppressed below the fundamental values.

From Proposition 4 we know that for high enough liquidity of the intermediary ( $I > I^*$ ), the manager behaves over aggressively by overinvesting in the risky asset (by setting  $\rho_R < \rho_R^{na}$ ) but underinvesting in the safer asset (by setting  $\rho_S > \rho_S^{na}$ ). It follows that for a high enough liquidity level of the intermediary,  $P_R^* > \bar{P}_R^*$ , but  $P_S^* < \bar{P}_S^*$ . We thus have the following corollary to Proposition 4.

**Corollary 3** *If the liquidity,  $I$ , of the intermediary is sufficiently high, then an asset price bubble is created in the asset price of the risky asset but concurrently an asset price “negative bubble” is created in the asset price of the safer asset.*

The formation of a bubble and negative bubble can also be illustrated by way of a four-quadrant diagram. In Fig. 4 we depict the mechanics behind the formation of a negative bubble. Quadrant I shows the relation between the risk of the safer project,  $1 - \theta$ , and the loan rate for the safer project,  $\rho_S$  as measured by line  $AA$ . Note that, in general, the higher the risk, the higher would be the equilibrium loan rate. The loan rate in turn determines the demand for loans and the volume of credit in the economy. The lower the loan rate, the higher is the amount of investment in the asset as is captured by line  $NN$  in Quadrant II. Quadrant III depicts the positive relation between investment and asset prices as captured by line  $BB$ . In general, an increase in investment pushes up asset demand, which in turn increases asset prices. Conversely, a reduction in investment reduces asset

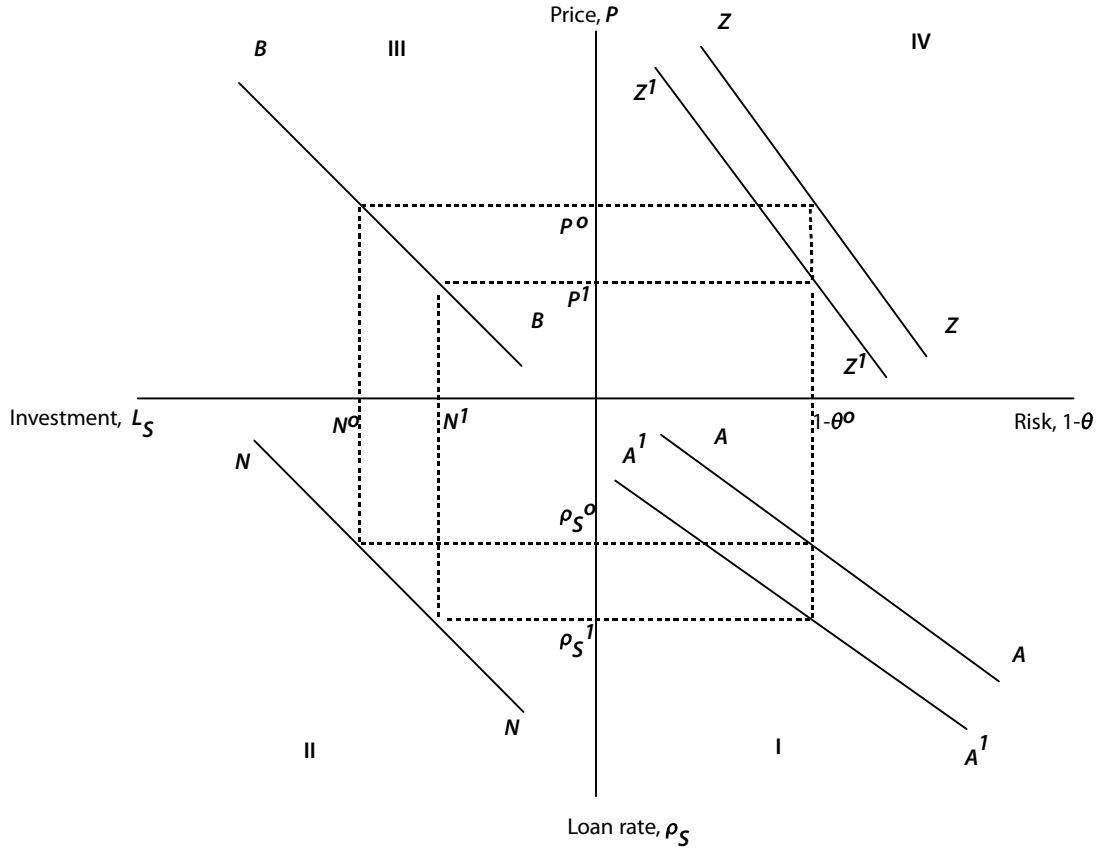


Figure 4: The mechanics of the formation of “negative bubbles”.

prices. Finally, Quadrant IV shows the relation between asset price and risk. The equilibrium relation between asset price and risk is derived by tracing the effects of risk on lending rate, which in turn influences investment, which subsequently affects the asset price. As expected, there is an inverse relation between asset price and risk as is captured by line  $ZZ$ , i.e. an increase in risk lowers the asset price and vice versa.

For example, if the risk of the safer project is given by  $1 - \theta^0$ , then as

shown in Quadrant I, the manager will set a loan rate of  $\rho_S^o$  as long as there are no agency problems. The amount of investment corresponding to a loan rate of  $\rho_S^o$  is given by  $N^o$  in Quadrant II. Given an investment of  $N^o$  the equilibrium asset price is given by  $P^o$  in Quadrant III. Tracing the relation between varying levels of risk and the corresponding asset price via loan rates and investment volumes, we can derive line  $ZZ$  in Quadrant IV which summarizes the negative relation between risk and asset price.

Let the line  $AA$  represent the fundamental relation between risk and loan rates, i.e. the relation that would prevail in the absence of any agency problems. Then, for any given level of risk, the fundamental asset price would be represented by the line  $ZZ$ . However, as discussed in Proposition 4, if the liquidity of the intermediary is sufficiently high then an agency problem is actuated whereby the manager crowds out investment in the safer asset so as to overinvest in the risky asset. In other words, for sufficiently high liquidity levels, the manager increases the loan rate for the safer asset for the same level of risk. This shifts the line  $AA$  to  $A^1A^1$  in Quadrant I and thus for the same level of risk the loan rate increases to  $\rho_S^1$ . An increase in the loan rate crowds out investment from  $N^o$  to  $N^1$  as shown in Quadrant II. The dampening of investment demand in turn reduces the asset price from  $P^o$  to  $P^1$  as can be seen in Quadrant III. Finally, Quadrant IV depicts that an increase in liquidity reduces the asset price from  $P^o$  to  $P^1$  for the same level of risk implying that the line  $ZZ$  shifts to the left to  $Z^1Z^1$ .

In short, once the agency problem is actuated, an asset price negative bubble is formed in the market for the safer asset. Using similar dynamics, we can show that the opposite happens in the market for the risky asset,

whereby an increase in the liquidity of the intermediary inflates the asset price of the risky asset thereby forming an asset price bubble in the market for the risky asset.<sup>12</sup>

Our analysis implies that a bubble in the market for an asset is accompanied by a negative bubble in the market for another asset. More specifically, a bubble in the market for the risky asset exists concurrently with a negative bubble in the market for the safer asset. Intuitively, overinvestment in one market crowds out investment in another market causing bubbles and negative bubbles to arise simultaneously.

Interestingly, the negative bubble is likely to arise in the market for the ‘safer’ assets rather than the ‘safest’ assets (for instance, cash equivalents like treasury bills). As discussed earlier, this effect arises due to the manager following his pecking order of first investing in the risky assets and then hoarding on to cash and cash equivalents so as to avoid the likelihood of liquidity shortfalls. Such a portfolio choice effectively dampens out the demand for the safer asset when the intermediary is flush with liquidity. Consequently, negative bubbles are more likely to arise in the market for safer or medium-risk assets whose liquidity risk is not as low as cash equivalents and at the same time offer lower returns to the manager relative to the higher bonuses received when investment is made in risky assets.

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<sup>12</sup>See Acharya and Naqvi (2012) for an illustration of the mechanics of the formation of a bubble.

## 4 Implications for leverage and risk-taking

We next examine the impact of leverage on the risk-taking incentives of risk averse managers in financial intermediaries. It is widely accepted that financial intermediaries are more vulnerable to runs if they are financed mostly by debt securities (such as demand deposit contracts) rather than equity.<sup>13</sup> To take this into account, we assume that debt investors are more likely to run on the intermediary as compared to equity investors. In other words, on average a higher fraction of debt investors withdraw their endowments in the interim period vis-a-vis equity investors. Alternatively, we can assume that a run by the equity investors is less costly since the intermediary is not obliged to prematurely liquidate its assets to accommodate the equity investors. On the other hand, a run by debt investors is more costly since contractually the intermediary is obliged to repay the debt investors and such contractual obligations can necessitate the premature liquidation of assets. Both of these assumptions lead to the same result.

Let us suppose a fraction  $\alpha$  of the investments are financed by equity,  $I^E$ , and the remaining fraction,  $1 - \alpha$ , is financed by debt,  $I^D$ .<sup>14</sup> Then an

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<sup>13</sup>See for example, Cochrane (2014), who notes that “if an institution is 95 percent financed by equity, there is little chance of bankruptcy, and thus little chance of a run.” This is because demand deposit investors have an incentive to run if they suspect that others will run also. On the other hand, equity holders have less incentive to run because they cannot force the intermediary into bankruptcy for failure to pay immediately. In general, if an institution has the right to delay payment then it is much harder for a run to develop.

<sup>14</sup>We do not consider the optimal capital structure of the intermediary since it is determined by tax issues, earnings dilution due to equity, monitoring costs of debt, regulatory

increase in the equity ratio,  $\alpha$ , would ex ante decrease the expected number of withdrawals in the interim period and/or reduce the cost of premature liquidation. We can then prove the following Proposition.

**Proposition 5** *As the fraction of investments financed by equity,  $\alpha$ , increases, the manager is more likely to overinvest in the risky asset and underinvest in the safer asset.*

The intuition behind Proposition 5 is as follows. If most of the intermediary's investments are financed by equity then the manager realizes that a fewer proportion of the investors will withdraw early and hence there will be more liquidity available in the interim period. Consequently, the probability of a liquidity shortfall (and thus an audit) will be lower and the manager is more likely to evade a penalty even if he acts overaggressively. This in turn increases the incentive of the manager to act overaggressively by overinvesting in the risky asset and underinvesting in the safer asset. Conversely, if the intermediary has a high leverage ratio,  $1 - \alpha$ , and is financed mostly by leverage then the intermediary is more vulnerable to runs in the interim period. Consequently, the probability of a liquidity shortfall in the interim period is higher and if the manager acts overaggressively he is more likely to be punished following an audit. Thus, the manager is less likely to act overaggressively if the intermediary has a high leverage ratio.

In other words, leverage acts as a disciplining device by constraining the risk-taking appetite of managers. This result is similar to the free cash flow constraints, etc. which are all outside the scope of this paper. Rather the focus of the paper is on how changes in leverage affect the risk-taking incentives of manager.

argument of Jensen (1986). In Jensen (1986) corporate managers follow their empire building aspirations in the presence of excessive free cash flow. In our case, intermediary managers have an incentive to increase the riskiness of their portfolios so as to increase their bonuses in the presence of abundant liquidity. In both cases, leverage can inhibit such liquidity induced agency problems. This result runs contrary to the philosophy behind capital adequacy requirement prescribed by regulators. Our analysis suggests that the presence of capital requirements can ex ante aggravate the agency problem inside intermediaries by incentivizing managers to act overaggressively. At the very least capital adequacy requirements are not a panacea and may have some unintended consequences in the form of riskier portfolios.

## 5 Discussion

Shin (2013) argues that it is useful to distinguish between two phases of “global liquidity”.<sup>15</sup> The first phase lasted from 2003-2008 and in this phase global banks were at the center transmitting loose financial conditions across borders via banking capital flows. The second phase of global liquidity started in 2010 and is very much relevant today. In this phase global banks have paved way to asset managers who are investing heavily in emerging market debt securities. The transmission of financial conditions from developed countries to emerging economies is now done largely via asset managers who are intrinsically “searching for yield”. The search for yield has led to a

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<sup>15</sup>CGFS (2011) defines global liquidity in broad terms as global financing conditions or “ease of financing”. The term is often used in connection with monetary policy spillovers from developed countries.



dramatic increase in investment by asset managers in emerging market debt securities. Consequently, there has been a large increase in the issuance of international debt securities to satisfy the corresponding demand.

We argue that the “searching for yield” behavior of asset managers is consistent with the findings of our model. The central banks of advanced countries have followed loose monetary policies since the 2008 financial crisis. The loose financial conditions have culminated in an increase in liquidity of financial intermediaries. Our model predicts that an influx of liquidity triggers an agency problem whereby managers of intermediaries search for yield and consequently overinvest in risky assets (e.g. emerging market debt securities) and underinvest in safer securities (e.g. investment grade debt securities of developed countries). The overinvestment in risky securities eventually leads to inflated prices of risky assets (translating into low yields for emerging market debt securities and other risky securities) and concurrently deflated prices for safer assets.<sup>16</sup>

Given the high demand for emerging market debt securities many firms in emerging markets have sought to finance their investments by issuing U.S. dollar denominated international debt securities. Indeed, McCauley, Upper, and Villar (2013) find that most of the offshore issuance by emerging market corporates have been in U.S. dollars. This implies that the transmission mechanism of the monetary policy of the Federal Reserve is not just limited to the U.S. but also affects the vulnerability of emerging economies to ex-

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<sup>16</sup>It is noteworthy that the yield on Spanish 10-year bond is currently the lowest since 1789. (See, for example, “The Fed’s Systemic-Risk Balancing Act - Wall Street Journal August 12, 2014) Perhaps, more suprisingly, Greece recently managed to sell 3 billion Euros worth of 5-year bonds at a yield of only 4.95%.

ternal shocks. Hence, if the Fed follows a loose monetary policy then the ensuing flow of liquidity into financial intermediaries affects the behavior of managers who in turn make substantial overinvestments in emerging market debt securities. Since such securities are largely issued in U.S. dollars, it makes the emerging market corporates much more sensitive to any changes in U.S. interest rates as well as to any fluctuations in the exchange rate of the local currency with respect to the U.S. dollar.

The danger is that such sensitivity of emerging economies to the monetary policy adopted by the Fed makes them very vulnerable to a tightening by the Federal Reserve. In the event of a tightening by the Federal Reserve the asset managers who are searching for yield are likely to reduce their exposure to emerging market debt by dumping such securities from their portfolios. Moreover, since the emerging market corporates are subject to a currency mismatch (given that a large chunk of their offshore debt is U.S. dollar denominated) their cost of debt will shoot up which will be exacerbated by an increase in the value of their debt as measured in their local currency. Furthermore, in the event of a corporate failure the domestic banking system is likely to come under pressure. In short, tighter monetary conditions will not just affect the U.S. financial markets but will also adversely impact the emerging economies. This potential of the asset managers to amplify economic shocks in the second phase of global liquidity is very similar to that of banks in the first phase of global liquidity.

Asset managers in the U.S. are not only hoarding onto emerging market debt securities but are also investing heavily in non-investment grade bonds while at the same time cutting down their positions in investment grade

securities. This is evident from the very low spread between non-investment grade and investment grade yields.<sup>17</sup> Using Federal Reserve Economic Data (FRED) in Fig. 5 we see that the spread between Moody's seasonal Baa Corporate yield and Aaa corporate yield has plummeted since the peak of the 2008 financial crisis.

## Appendix: Proofs

### *Proof of Proposition 1*

The participation constraint of the intermediary is binding because otherwise the intermediary can increase its expected profit by slightly reducing  $\rho_I$ . Thus,  $\rho_I^*$  is given by the solution to

$$E(\tilde{x}) + (1 - E(\tilde{x})) \left[ \theta \rho_I + (1 - \theta) \frac{E[\max(C - \tilde{x}I, 0)]}{(1 - E(\tilde{x}))I} \right] = \bar{u}. \quad (40)$$

Solving for  $\rho_I^*$  gives us Eq. (11).

From Eq. (7) we can solve for  $C$  which gives us

$$C = D - L(\rho_R) + L(\rho_S). \quad (41)$$

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<sup>17</sup>According to a recent report of the Wall Street Journal, "Interest rates globally - and especially in Europe - are historically low...And risk appetite is big... Junk bond yields overall are an average 3.75 percentage points higher than those for U.S. Treasury debt - the lowest gap since October 2007, according to Barclays... Junk-bond borrowers can typically get away with only small bond offerings, but appetite is picking up after plunging in the financial crisis. So far this year, corporate junk borrowers worldwide have issued more than \$139 billion in bonds, excluding financial issuers, according to data provider Dealogic. At this point in 2008, the sum was just \$15 billion. ...And Europe's debt crisis has forced banks to reduce lending, so many companies have turned to bond investors to fill the gap." - Record Sale for Junk Bonds, Wall Street Journal, April 23, 2014.



Figure 5: The spread between Moody's Seasoned Baa Corporate Bond Yield and Moody's Seasoned Aaa Corporate Bond Yield. Source: St. Louis Federal Reserve

Substituting  $\rho_I^*$  and  $C$  in the intermediary's objective function gives us the following unconstrained maximization problem:

$$\max_{\rho_R, \rho_S} \Pi = \pi - E[\Psi | e = e_H] \quad (42)$$

where  $\pi$  is given by Eq. (8) and  $\Psi$  is given by Eq. (4).

Assuming that  $\Pi$  is quasiconcave in  $\rho_i$  for  $i = R, S$  the first-order condition (FOC) with respect to  $\rho_S$  is given by

$$\begin{aligned} \frac{\partial \Pi}{\partial \rho_S} &= \theta L(\rho_S) - \theta \Pr[\tilde{x}I \leq C] L'(\rho_S) + \theta \rho_S L'(\rho_S) \\ &\quad - \rho_S^p \Pr[\tilde{x}I > C] L'(\rho_S) - \theta I(1 - E(\tilde{x})) \frac{\partial \rho_I^*}{\partial \rho_S} = 0. \end{aligned} \quad (43)$$

Noting that  $\partial \rho_I^* / \partial \rho_S = (1 - \theta) \Pr[\tilde{x}I \leq C] L'(\rho_S) / \theta I(1 - E(\tilde{x}))$  and solving for  $\rho_S$  after some simplification we get Eq. (9).

Next, taking the FOC with respect to  $\rho_R$ , we get

$$\begin{aligned} \frac{\partial \Pi}{\partial \rho_R} &= \theta p L(\rho_R) - \theta \Pr[\tilde{x}I \leq C] L'(\rho_R) + \theta p \rho_R L'(\rho_R) \\ &\quad - \rho_S^p \Pr[C < \tilde{x}I \leq C + L(\rho_S)] L'(\rho_R) \\ &\quad - \rho_R^p \Pr[\tilde{x}I > C + L(\rho_S)] L'(\rho_R) - \theta I(1 - E(\tilde{x})) \frac{\partial \rho_I^*}{\partial \rho_R} = 0. \end{aligned} \quad (44)$$

Noting that  $\partial \rho_I^* / \partial \rho_R = (1 - \theta) \Pr[\tilde{x}I \leq C] L'(\rho_R) / \theta I(1 - E(\tilde{x}))$  and solving for  $\rho_R$  after some simplification we get Eq. (10).

Finally, substituting  $\rho_R^*$  and  $\rho_S^*$  in Eq. (41) we get  $C^*$  as given by Eq. (12). Q.E.D.

#### *Proof of Proposition 1*

From the FOC (43), if we solve for  $\rho_S^*$  directly without exploiting the definition of  $\eta_S$  we get the following expression for the return on safer loans:

$$\rho_S^* = \frac{1}{\theta} - \frac{L(\rho_S)}{L'(\rho_S)} + \frac{(\rho_S^p - 1) \Pr(\tilde{x}I \geq C^*)}{\theta}. \quad (45)$$

Taking the partial derivative of the above expression w.r.t.  $\theta$  we get:

$$\frac{\partial \rho_S^*}{\partial \theta} = -\frac{1 + (\rho_S^p - 1) \Pr(\tilde{x}I \geq C^*)}{\theta^2} < 0 \quad (46)$$

since  $\rho_S^p > \rho_S > 1$ , which proves the risk effect for the safer loan.

Next note that  $\partial \Pr(\tilde{x}I \geq C) / \partial I < 0$ , i.e. an increase in financial intermediary's liquidity (investment funds) lowers the probability of liquidity shortfalls since  $C = D - L_R - L_S$ . Then taking the partial derivative of (45) w.r.t.  $1 - F(C) = \Pr(\tilde{x}I \geq C)$  we get:

$$\frac{\partial \rho_S^*}{\partial [1 - F(C)]} = \frac{\rho_S^p - 1}{\theta} > 0. \quad (47)$$

Hence  $\frac{\partial \rho_S^*}{\partial I} = \frac{\partial \rho_S^*}{\partial [1 - F(C)]} \frac{\partial [1 - F(C)]}{\partial I} < 0$ , which proves the liquidity effect for the safer loan.

Similarly, from the FOC (44), if we solve for  $\rho_R^*$  directly without exploiting the definition of  $\eta_R$  we get the following expression for the return on risky loans:

$$\rho_R^* = -\frac{L(\rho_R)}{L'(\rho_R)} + \frac{\Pr(\tilde{x}I \leq C) + \rho_S^p \Pr(C < \tilde{x}I \leq C + L_S) + \rho_R^p \Pr(\tilde{x}I > C + L_S)}{\theta p}. \quad (48)$$

Taking the partial derivative of the above expression w.r.t.  $\theta$  we get:

$$\frac{\partial \rho_R^*}{\partial \theta} = -\frac{p [\Pr(\tilde{x}I \leq C) + \rho_S^p \Pr(C < \tilde{x}I \leq C + L_S) + \rho_R^p \Pr(\tilde{x}I > C + L_S)]}{(\theta p)^2} < 0, \quad (49)$$

which proves that an increase in macroeconomic risk,  $1 - \theta$ , increases the equilibrium lending rate for the risky project, ceteris paribus.

Similarly, taking the partial derivative of Eq. (48) w.r.t.  $p$  we get:

$$\frac{\partial \rho_R^*}{\partial p} = -\frac{\theta [\Pr(\tilde{x}I \leq C) + \rho_S^p \Pr(C < \tilde{x}I \leq C + L_S) + \rho_R^p \Pr(\tilde{x}I > C + L_S)]}{(\theta p)^2} < 0, \quad (50)$$

which proves that an increase in specific risk,  $1 - p$ , increases the equilibrium lending rate for the risky project, ceteris paribus.

Finally, taking the partial derivative of Eq. (48) w.r.t.  $I$  we get:

$$\begin{aligned} \frac{\partial \rho_R^*}{\partial I} &= \left[ f \left( \frac{C}{I} \right) \frac{L(\rho_S) + L(\rho_R)}{I^2} \right] \left[ \frac{1}{\theta p} - \frac{\rho_S^p}{\theta p} \right] \\ &+ \left[ f \left( \frac{C + L(\rho_S)}{I} \right) \frac{L(\rho_R)}{I^2} \right] \left[ \frac{\rho_S^p}{\theta p} - \frac{\rho_R^p}{\theta p} \right] < 0, \end{aligned} \quad (51)$$

since  $\rho_R^p > \rho_S^p > 1$ , which proves the liquidity effect for the risky loan. Q.E.D.

*Proof of Proposition 2*

Let  $\mu_1, \mu_2, \mu_3$  denote the Lagrange multipliers for constraints (14), (15), and (16). Taking the FOC with respect to  $b_S$  the following condition is satisfied at every  $L_R$ :

$$\frac{1}{\int_I \int_x [(1 - \phi) v'(b) + \phi v'(b - \psi)] f(x) j(I) dx dI} = \mu_1, \quad (52)$$

where  $j(I)$  is the density function of intermediary's liquidity,  $I$ . Since the RHS in Eq. (52) is constant it follows that  $v'(\cdot)$  on the LHS is constant. If the manager is strictly risk averse (so that  $v'(w)$  is strictly decreasing in  $w$ ), the implication of condition (52) is that  $b_S$  is constant.

Next, taking the FOC with respect to  $b_R$  the following condition is satisfied at every  $L_R$ :

$$\frac{1}{\int_I \int_x [(1 - \phi) v'(b) + \phi v'(b - \psi)] f(x) j(I) dx dI} = \mu_1 + \mu_2 \left[ 1 - \frac{g(L(\rho_R) | e^L)}{g(L(\rho_R) | e^H)} \right], \quad (53)$$

where  $g(L(\rho_R)|e) > 0$  is the density function of loans conditional on effort. As is common in the literature, we then invoke the monotone likelihood ratio property (MLRP), i.e.,  $[g(L(\rho_R)|e^L)/g(L(\rho_R)|e^H)]$  is decreasing in  $L_R$ . This means that, as risky loans increase, the likelihood of getting a given level of risky loans and profits if effort is  $e^H$ , relative to the likelihood if effort is  $e^L$ , must increase. Hence, an increase in  $L_R$  increases the right-hand side (RHS) of Eq. (53). It follows that the left-hand side (LHS) is increasing in  $L_R$  and the denominator of the LHS is decreasing in  $L_R$ . The denominator of LHS will be decreasing in  $L_R$  if, and only if,  $v'(\cdot)$  is decreasing in  $L_R$ . Note, however, that  $\psi = \min(\bar{\psi}, \gamma\Psi)$  is increasing in  $L_R$ . This is clear once we rewrite Eq. (4) by substituting for  $C = I - L_R - L_S$  to get the following expression for  $\Psi$ :

$$\Psi = \begin{cases} \rho_S^p [L_R + L_S - I(1-x)] & \text{if } 0 < \ell \leq L_S \\ \rho_S^p L_S + \rho_R^p [L_R - I(1-x)] & \text{if } \ell > L_S \end{cases} \quad (54)$$

Since  $\psi$  is increasing in  $L_R$  and given that  $v'' < 0$ , it follows that the denominator of the LHS is decreasing in  $L_R$  if, and only if, managerial bonuses,  $b_R$ , are monotonically increasing in  $L_R$ .

Next, taking the FOC with respect to  $\psi$ , the following condition is satisfied for every  $L_R$ :

$$\begin{aligned} \int_I \int_x \left[ 1 - \mu_1 v'(b - \psi) - \mu_2 v'(b - \psi) \left( 1 - \frac{g(L(\rho_R)|e^L)}{g(L(\rho_R)|e^H)} \right) \right] \phi g(L(\rho_R)|e) f(x) j(I) dx dI \\ = \mu_3. \end{aligned} \quad (55)$$

Because constraint (16) is binding, it follows that  $\mu_3 > 0$ . Thus, the following



condition is satisfied:

$$\left[ 1 - \mu_1 v'(\cdot) - \mu_2 v'(\cdot) \left( 1 - \frac{g(L(\rho_R)|e^L)}{g(L(\rho_R)|e^H)} \right) \right] > 0. \quad (56)$$

Finally, taking the FOC with respect to  $\phi$ , the following condition is satisfied for every  $L_R$ :

$$\begin{aligned} & \int_I \int_x \psi g(L(\rho_R)|e^H) f(x) j(I) dx dI - z \\ & + \mu_1 \int_I \int_x [-v(b) + v(b - \psi)] g(L(\rho_R)|e^H) f(x) j(I) dx dI \\ & + \mu_2 \int_I \int_x [-v(b) + v(b - \psi)] [g(L(\rho_R)|e^H) - g(L(\rho_R)|e^L)] f(x) j(I) dx dI \\ & + (\mu_4 - \mu_5) = 0, \end{aligned} \quad (57)$$

where  $\mu_4$  and  $\mu_5$  correspond to the Lagrange multipliers for the constraints  $\phi \geq 0$  and  $\phi \leq 1$ , respectively. An audit takes place if, and only if,

$$\begin{aligned} k(\ell) &= \int_I \int_x \psi g(L(\rho_R)|e^H) f(x) j(I) dx dI - z \\ & + \mu_1 \int_I \int_x [-v(b) + v(b - \psi)] g(L(\rho_R)|e^H) f(x) j(I) dx dI \\ & + \mu_2 \int_I \int_x [-v(b) + v(b - \psi)] [g(L(\rho_R)|e^H) - g(L(\rho_R)|e^L)] f(x) j(I) dx dI > 0. \end{aligned} \quad (58)$$

This is because, if  $k(\ell) > 0$ , it implies that  $\mu_5 > \mu_4$ . But  $\mu_5 > \mu_4$  if, and only if, the constraint  $\phi \leq 1$  is binding as a binding constraint implies that  $\mu_5 > 0$  but  $\mu_4 = 0$ . This would be the case if, and only if,  $\phi = 1$ . It follows

that  $\phi = 1$  if  $k(\ell) > 0$  and  $\phi = 0$  otherwise. Let  $\ell^*$  denote the threshold such that  $k(\ell^*) = 0$ . To prove that it is optimal to audit if, and only if,  $\ell > \ell^*$ , it would suffice to show that  $k'(\ell)$  is strictly increasing in  $\ell$ .

Taking the derivative of  $k(\ell)$  with respect to  $\ell$  after some simplification we get

$$k'(\ell) = \int_I \int_x \left[ 1 - \mu_1 v'(b - \psi) - \mu_2 v'(b - \psi) \left( 1 - \frac{g(L(\rho_R) | e^L)}{g(L(\rho_R) | e^H)} \right) \right] g(L(\rho_R) | e^H) \psi'(\ell) dF dJ, \quad (59)$$

where  $F$  and  $J$  represent the distribution functions of  $x$  and  $I$ , respectively. Because  $\psi'(\ell) > 0$  and given condition (56), it follows that  $k'(\ell) > 0$ . Q.E.D.

### *Proof of Proposition 3*

As before the participation constraint is binding from which we can solve for  $\rho_I^{na}$ . Also from the budget constraint, we have  $C = I - L(\rho_R) - L(\rho_S)$ . Substituting  $\rho_I^{na}$  and  $C$  in  $\pi$  we need to solve for an unconstrained maximization problem. Taking the FOC with respect to  $\rho_i$  and solving for  $\rho_S^{na}$  and  $\rho_R^{na}$  we get

$$\begin{aligned} \rho_S^{na} &= \frac{\Pr[(\tilde{x}I \leq I) | e^H] + \rho_S^p \Pr[(\tilde{x}I > C^*) | e^H]}{\theta \left( 1 - \frac{1}{\bar{\eta}_S} \right)} + \frac{\frac{\partial \hat{E}[b+z | e=e^H]}{\partial \rho_S}}{\theta L'(\rho_S)}, \quad (60) \\ \rho_R^{na} &= \frac{\Pr[(\tilde{x}I \leq C) | e^H] + \rho_S^p \Pr[(C < \tilde{x}I \leq C + L_S) | e^H] + \rho_R^p \Pr[(\tilde{x}I > C + L_S) | e^H]}{\theta p \left( 1 - \frac{1}{\bar{\eta}_R} \right)} \\ &\quad + \frac{\frac{\partial \hat{E}[b+z | e=e^H]}{\partial \rho_R}}{\theta \frac{\partial \hat{E}[L | e=e^H]}{\partial \rho_R}}, \quad (61) \end{aligned}$$

where  $\bar{\eta}_S = -\rho_S L'(\rho_S) / L_S > 0$  and  $\bar{\eta}_R = -\rho_R \frac{\partial \hat{E}[L(\rho_R) | e=e^H] / \partial \rho_R}{\hat{E}[L(\rho_R) | e=e^H]}$ . In the case of symmetric information,  $\hat{E}[L(\rho_R) | e = e^H] = L(\rho_R) | e = e^H$  since

risky loans are non-stochastic. It follows that  $\bar{\eta}_R = \eta_R$  with symmetric information. Noting that the first term on the RHS of Eqs. (60) and (61) is  $r_S^*$  and  $r_R^*$  respectively we get expressions (22) and (23). Next note that

$$\frac{\partial \hat{E} [b + z|e = e^H]}{\partial \rho_i} = \frac{\partial \hat{E} [b + z|e = e^H]}{\partial L_i} \frac{\partial L_i}{\partial \rho_i} < 0 \quad (62)$$

for  $i = R, S$  since bonuses,  $b$ , are increasing in loan volume; audit costs ( $z$ ) are increasing in loan volume (of both safer and risky loans) since an increase in loan volume increases the probability of liquidity shortfalls thereby increasing the expected audit costs ( $z$ ); while  $\partial L_i / \partial \rho_i < 0$ . Finally noting that  $L'(\rho_S) < 0$  and  $\partial \hat{E} [L_R|e = e^H] / \partial \rho_R < 0$  it follows that the second term on the RHS of (62) is positive and thus  $\rho_S^{na} > \rho_S^*$  and  $\rho_R^{na} > \rho_R^*$ . Q.E.D.

*Proof of Proposition 4*

We can rewrite the manager's problem as follows:

$$\max_{\rho_R, \rho_S, C} \int_{L_R} \int_x v \left( \left[ b(L(\rho_R)) + b_S - \tilde{\psi} \right] | e = e^H \right) f(x) g(L_R | e = e^H) dx dL_R - e^H \quad (63)$$

subject to

$$L(\rho_R) + L(\rho_S) + C = I, \quad (64)$$

and

$$L(\rho_S) \geq \underline{L}_S^j \quad \forall \theta^j. \quad (65)$$

where  $\underline{L}_S^j$  is decreasing in  $\theta^j$ . Taking the FOC with respect to  $\rho_R$  we get

$$\int_{L_R} \int_x v'(\cdot) \left[ b'_R(L_R) L'_R(r_R^{a*}) - \frac{\partial \tilde{\psi}}{\partial \rho_R} \right] f(x) g(L_R | e = e^H) dx dL_R - \lambda_1 L'_R = 0 \quad (66)$$

where  $\lambda_1$  is the Lagrange multiplier for constraint (64). Taking the FOC with respect to  $\rho_S$  we get

$$\int_{L_R} \int_x v'(\cdot) \left[ -\frac{\partial \tilde{\psi}}{\partial \rho_S} \right] f(x) g(L_R | e = e^H) dx dL_R - \lambda_1 L'_S + \lambda_2 L'_S = 0 \quad (67)$$

where  $\lambda_2$  is the Lagrange multiplier for constraint (65). Finally, taking the FOC with respect to  $C$  we get

$$\int_{L_R} \int_x v'(\cdot) \left[ -\frac{\partial \tilde{\psi}}{\partial C} \right] f(x) g(L_R | e = e^H) dx dL_R - \lambda_1 = 0. \quad (68)$$

The first term in FOC (67) is positive. This is because  $\frac{\partial \tilde{\psi}}{\partial \rho_S} = \frac{\partial \tilde{\psi}}{\partial L_S} L'_S$ . An increase in  $L_S$  reduces cash holdings and hence increases  $\tilde{\psi}$ . It follows that  $\partial \tilde{\psi} / \partial L_S > 0$ . Given that  $L'_S < 0$  it follows that  $\partial \tilde{\psi} / \partial \rho_S < 0$  and thus the first term is positive. Since the budget constraint (64) is binding, the lagrange multiplier,  $\lambda_1 > 0$ . Since  $L'_S < 0$ , the second term in (67) is also positive. It follows that in order for the FOC to be satisfied,  $\lambda_2 > 0$ . This implies that the second constraint (65) is also binding and hence  $L(\rho_S) = L_S^j \forall \theta^j$ .

In FOC (68) the first term is positive since  $v'(\cdot) > 0$  and  $\partial \tilde{\psi} / \partial C < 0$  given that an increase in cash holdings lowers the penalty costs. It follows from Eq. (68) that  $\lambda_1$  is given by:

$$\lambda_1 = \int_{L_R} \int_x v'(\cdot) \left[ -\frac{\partial \tilde{\psi}}{\partial C} \right] f(x) g(L_R | e = e^H) dx dL_R > 0. \quad (69)$$

Substituting Eq. (69) in FOC (66) we get the following condition:

$$\begin{aligned}
& \int_{L_R} \int_x v'(\cdot) \left[ b'_R(L_R) L'_R(\rho_R^{a*}) - \frac{\partial \tilde{\psi}}{\partial r_R} \right] f(x) g(L_R|e = e^H) dx dL_R (70) \\
& = \left[ \int_{L_R} \int_x v'(\cdot) \left[ -\frac{\partial \tilde{\psi}}{\partial C} \right] f(x) g(L_R|e = e^H) dx dL_R \right] L'_R.
\end{aligned}$$

Eq. (70) says that at the optimum the manager chooses the volume of risky loans,  $L_R$ , such that the net marginal benefit of an incremental loan (given by the LHS of Eq. (70) just equals the marginal costs (given by the RHS of Eq. (70)). In other words, at the optimum the net marginal benefit of issuing risky loans (given by the expected increase in managerial commissions minus the expected increase in penalties) just equals the marginal cost (since the same amount could have been invested in liquid cash reserves thereby reducing the expected penalty cost for the manager).

The manager behaves overaggressively if, and only if, his expected utility from acting overaggressively exceeds his expected utility from not acting overaggressively. More formally, this is true if, and only if, the following expression is positive:

$$\begin{aligned}
\Delta \Pi_m &= \int_{L_R} \int_x v \left( b(L_R^a) + b_S - \tilde{\psi}|e = e^H \right) f(x) g(L_R|e = e^H) dx dL_R \\
&\quad - \int_{L_R} v \left( b(L_R^{na}) + b_S|e = e^H \right) g(L_R|e = e^H) dL_R, \tag{71}
\end{aligned}$$

where  $L_R^a$  denotes the loan volume when the manager acts overaggressively;  $L_R^{na}$  denotes the loan volume when the manager does not act overaggressively; and  $\Delta \Pi_m$  denotes the expected utility of the manager from acting overaggressively minus the expected utility from not acting overaggressively conditional

on high effort. In Eq. (71)  $L_R^a > L_R^{na}$  and thus  $\tilde{\psi} > 0$ . If,  $L_R^a = L_R^{na}$ , then there's no agency problem and thus  $\Delta\Pi_m = 0$ . We next show that  $\Delta\Pi_m > 0$  for sufficiently large  $I$ .

Adding and subtracting  $\int_{L_R} v(b(L_R^a) + b_S|e = e^H) g(L_R|e = e^H) dL_R$  to Eq. (71) yields

$$\begin{aligned} \Delta\Pi_m &= \int_{L_R} v(b(L_R^a) + b_S|e = e^H) g(L_R|e = e^H) dL_R \\ &\quad - \int_{L_R} v(b(L_R^{na}) + b_S|e = e^H) g(L_R|e = e^H) dL_R - c, \end{aligned} \quad (72)$$

where

$$\begin{aligned} c &\equiv \int_{L_R} v(b(L_R^a) + b_S|e = e^H) g(L_R|e = e^H) dL_R \\ &\quad - \int_{L_R} \int_x v\left(\left[b(L_R^a) + b_S - \tilde{\psi}\right] | e = e^H\right) f(x) g(L_R|e = e^H) dx dL_R \end{aligned} \quad (73)$$

The first term in Eq. (72) is positive because  $L_R^a > L_R^{na}$  and  $v'(\cdot) > 0$ . Hence,  $\Delta\Pi_m > 0$  as long as  $c$  is small enough. It can then be shown that  $c$  is decreasing in  $I$  and, for high enough  $I$ ,  $\Delta\Pi_m > 0$ . Thus, to prove the proposition it would suffice to show that  $c$  is decreasing in  $I$ .

Note that

$$\begin{aligned} &\int_{L_R} \int_x v\left(\left[b(L_R^a) + b_S - \tilde{\psi}\right] | e = e^H\right) f(x) g(L_R|e = e^H) dx dL_R \quad (74) \\ &= \int_{L_R} \int_x (1 - \phi) v(b(L_R^a) + b_S|e = e^H) f(x) g(L_R|e = e^H) dx dL_R \\ &\quad + \int_{L_R} \int_x \phi v(b(L_R^a) + b_S - \psi|e = e^H) f(x) g(L_R|e = e^H) dx dL_R, \end{aligned}$$

where  $\phi = \Pr(\ell > \ell^*)$ .

Substituting Eq. (74) in Eq. (73) and taking the partial derivative of Eq. (73) with respect to  $I$  after some simplification yields

$$\begin{aligned} & \int_{L_R} \int_x \phi [v'(b(L_R^a) + b_S) - v'(b(L_R^a) + b_S - \psi|\cdot)] \left[ \frac{\partial b}{\partial I} \right] f(x) g(L_R|\cdot) dx dL_R \\ & + \int_{L_R} \int_x \phi [v'(b(L_R^a) + b_S - \psi|\cdot)] \left[ \frac{\partial \psi}{\partial I} \right] f(x) g(L_R|\cdot) dx dL_R \\ & + \int_{L_R} \int_x \left( \frac{\partial \phi}{\partial I} \right) [v(b(L_R^a) + b_S|\cdot) - v(b(L_R^a) + b_S - \psi|e = e^H)] f(x) g(L_R|\cdot) dx dL_R. \end{aligned} \quad (75)$$

In the first term, note that  $\frac{\partial b_R}{\partial I} = \frac{\partial b_R}{\partial L_R} \frac{\partial L_R}{\partial I}$ . As proved in Proposition 2  $\frac{\partial b_R}{\partial L_R} > 0$  because bonuses are increasing in loan volume. Further,  $\frac{\partial L_R}{\partial I} = \frac{\partial L_R}{\partial \rho_R} \frac{\partial \rho_R}{\partial I}$ . We know  $\frac{\partial L_R}{\partial \rho_R} < 0$  and  $\frac{\partial \rho_R}{\partial I} < 0$  because the loan rate is decreasing in liquidity. Hence,  $\frac{\partial L_R}{\partial I} > 0$  and thus  $\frac{\partial b_R}{\partial I} > 0$ . Furthermore,

$$[v'(b(L_R^a) + b_S) - v'(b(L_R^a) + b_S - \psi|\cdot)] < 0 \quad (76)$$

because  $v''(\cdot) < 0$ . Hence, the first term in expression (75) is negative. Next note that  $\frac{\partial \psi}{\partial I} < 0$  because the penalty  $\psi = \min(\bar{\psi}, \gamma\Psi)$  is increasing in  $\ell$ . Hence, the second term in (75) is negative. Finally, note that  $\frac{\partial \phi}{\partial I} < 0$ . This is because the ex ante audit probability is given by  $\phi = \Pr(\ell > \ell^*)$ , where  $\ell = \max[xI - C, 0] = \max[L_R + L_S - (1 - x)I, 0]$  given that  $C = D - L_R - L_S$ . Because  $\ell$  is decreasing in  $I$ , it follows that the audit probability,  $\phi$ , is decreasing in  $I$ . Furthermore,  $\{v(b|\cdot) - [v(b - \psi|\cdot)]\} > 0$  because  $b > b - \psi$  and because  $v'(\cdot) > 0$ . Hence, the third term in (75) is also negative. Q.E.D.

#### *Proof of Proposition 5*

In order to show that an increase in the equity ratio,  $\alpha$ , increases the likelihood that the manager will act overaggressively, it would suffice to show

that  $\Delta\Pi_m$  as given by Eq. (71) is increasing in  $\alpha$ . We can rewrite  $\Delta\Pi_m$  as follows:

$$\begin{aligned}\Delta\Pi_m &= \int_{L_R} \int_x (1 - \phi) v (b_R^a + b_S) f(x) g(L_R|\cdot) dx dL_R \\ &\quad + \int_{L_R} \int_x \phi v (b_R^a + b_S - \psi) f(x) g(L_R|\cdot) dx dL_R \\ &\quad - \int_{L_R} v (b_R^{na} + b_S) f(x) g(L_R|\cdot) dL_R,\end{aligned}\tag{77}$$

where for brevity  $b_R^a = b(L_R^a)$  and  $b_R^{na} = b(L_R^{na})$ . Taking the derivative of  $\Delta\Pi_m$  with respect to  $\alpha$  we get

$$\begin{aligned}\frac{\partial\Delta\Pi_m}{\partial\alpha} &= \int_{L_R} \int_x v' (b_R^a + b_S) \frac{\partial b_R^a}{\partial\alpha} f(x) g(L_R|\cdot) dx dL_R \\ &\quad - \int_{L_R} \int_x v (b_R^a + b_S) \frac{\partial\phi}{\partial\alpha} f(x) g(L_R|\cdot) dx dL_R \\ &\quad - \int_{L_R} \int_x \phi v' (b_R^a + b_S) \frac{\partial b_R^a}{\partial\alpha} f(x) g(L_R|\cdot) dx dL_R \\ &\quad + \int_{L_R} \int_x v (b_R^a + b_S - \psi) \frac{\partial\phi}{\partial\alpha} f(x) g(L_R|\cdot) dx dL_R \\ &\quad + \int_{L_R} \int_x \phi v' (b_R^a + b_S - \psi) \left[ \frac{\partial b_R^a}{\partial\alpha} - \frac{\partial\psi}{\partial\alpha} \right] f(x) g(L_R|\cdot) dx dL_R \\ &\quad - \int_{L_R} v' (b_R^{na} + b_S) \frac{\partial b_R^{na}}{\partial\alpha} f(x) g(L_R|\cdot) dL_R\end{aligned}\tag{78}$$



Simplifying we get

$$\begin{aligned}
\frac{\partial \Delta \Pi_m}{\partial \alpha} = & \int_{L_R} \int_x v' (b_R^a + b_S) \frac{\partial b_R^a}{\partial \alpha} (1 - \phi) f(x) g(L_R|\cdot) dx dL_R \\
& + \int_{L_R} \int_x [-v (b_R^a + b_S) + v (b_R^a + b_S - \psi)] \frac{\partial \phi}{\partial \alpha} f(x) g(L_R|\cdot) dx dL_R \\
& + \int_{L_R} \int_x \phi \left[ v' (b_R^a + b_S - \psi) \frac{\partial b_R^a}{\partial \alpha} - v' (b_R^{na} + b_S) \frac{\partial b_R^{na}}{\partial \alpha} \right] f(x) g(L_R|\cdot) dx dL_R \\
& - \int_{L_R} \int_x \phi v' (b_R^a + b_S - \psi) \frac{\partial \psi}{\partial \alpha} f(x) g(L_R|\cdot) dx dL_R
\end{aligned} \tag{79}$$

Note that  $\frac{\partial b_R^a}{\partial \alpha} = \frac{\partial b_R^a}{\partial L_R^a} \frac{\partial L_R^a}{\partial \rho_R^a} \frac{\partial \rho_R^a}{\partial \alpha}$ . Recall that  $b_R$  is increasing in  $L_R$  (as shown in Proposition 2). Also,  $\frac{\partial \rho_R^a}{\partial \alpha} < 0$  since an increase in  $\alpha$  reduces the liquidity risk by lowering  $E(\tilde{x})$  which in turn decreases the loan rate  $\rho_R^a$  (since the intermediary passes some of the benefits of lowered liquidity risk to the borrowers). Since  $\frac{\partial L_R^a}{\partial \rho_R^a} < 0$  it follows that  $\frac{\partial b_R^a}{\partial \alpha} > 0$ . Thus the first term in Eq. (79) is positive.  $\frac{\partial \phi}{\partial \alpha} < 0$  since an increase in  $\alpha$  lowers the probability of a sufficiently big liquidity shortfall to trigger an audit. Since  $v'(\cdot) > 0$  it follows that  $v(b_R^a + b_S) > v(b_R^a + b_S - \psi)$ . Thus the second term in Eq. (79) is also positive. Next note that  $\left| \frac{\partial \rho_R^a}{\partial \alpha} \right| > \left| \frac{\partial \rho_R^{na}}{\partial \alpha} \right|$ . An increase in the equity ratio decreases both  $\rho_R^a$  and  $\rho_R^{na}$  because of the reduction in liquidity risk. However, an increase in the equity ratio also decreases  $\phi$  which in turn further decreases  $\rho_R^a$  due to the agency effect. Formally,  $\frac{\partial \rho_R^a}{\partial \alpha} = \frac{\partial \rho_R^a}{\partial E(\tilde{x})} \frac{\partial E(\tilde{x})}{\partial \alpha} + \frac{\partial \rho_R^a}{\partial \phi} \frac{\partial \phi}{\partial \alpha}$  where the agency effect in the second term reinforces the effect in the first term. In the absence of an agency problem, the agency effect is zero and thus  $\left| \frac{\partial \rho_R^a}{\partial \alpha} \right| > \left| \frac{\partial \rho_R^{na}}{\partial \alpha} \right|$ . Since  $\frac{\partial b_R}{\partial \alpha} = \frac{\partial b_R}{\partial L_R} \frac{\partial L_R}{\partial \rho_R} \frac{\partial \rho_R}{\partial \alpha}$  it follows that  $\frac{\partial b_R^a}{\partial \alpha} > \frac{\partial b_R^{na}}{\partial \alpha}$ . Noting that  $b_R^a + b_S - \psi < b_R^{na} + b_S$  and since  $v''(\cdot) < 0$  it follows that the third

term is also positive. Finally, the last term is positive since  $\frac{\partial \psi}{\partial \alpha} < 0$  since an increase in the equity ratio decreases liquidity risk and thus decreases the managerial penalty. It follows that  $\frac{\partial \Delta \Pi_m}{\partial \alpha} > 0$ . Q.E.D.

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