Firm Investment Decisions under Hyperbolic Discounting

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Abstract

This paper constructs a model of corporate investment decisions under hyperbolic discounting of present values. We show that such a firm faces an underinvestment problem: its' equilibrium investment levels are lower than optimal in that there exists another feasible investment plan that improves all periods' present values. An outside authority, such as the government, has the ability to address the underinvestment problem by implementing Pareto-improving policies. Specifically, we show that imposing revenue-neutral dividend taxes or investment subsidies can overcome the firm's underinvestment problem and consequently increase all periods' present value of dividends. Lastly, under a multi-period extension with Cobb-Douglas return functions, we show that the magnitude of underinvestment induced by present bias increases as time elapses.

Keywords: Present bias; hyperbolic discounting; corporate investment; underinvestment; dividend taxation; investment subsidy.

JEL classification: D03; D21; D92; E22; G02; G31; G38.

1 Introduction

The notion of short-termism behavior among corporations has been widely discussed. Practitioners of finance and policymakers often cite short-termism as a major constraint on value-enhancing corporate investment projects (e.g. Graham et al. 2005). Short-termism also features prominently in public policy debates on corporate taxation. However, despite the wide attention received, theoretical underpinnings for the linkage between short-termism and corporate investment remain extremely sparse. This paper attempts to fill in the gap by constructing a theoretical model of corporate investment decisions under short-termism and analyze its associated policy implications.

In particular, we introduce hyperbolic discounting to corporate investment decisions. We present a finite-period model under which the firm makes an investment decision in each period to maximize the present value of its dividend stream. The firm invests in one project that yields return in the final period. We show that a firm exhibiting hyperbolic discounting preferences faces an underinvestment problem, i.e. there exists another feasible investment plan that improves all periods' present values. Therefore, Pareto-improving policies by an outside authority, such as the government, may be justified. In the second part of this paper, we show that adopting revenue-neutral dividend taxes or investment subsidies can mitigate the firm's underinvestment problem and thus increase all periods' present value of dividends.

This paper is related to a few strands of literature. First, it directly addresses the issue of short-termism in economics. Experimental and introspective evidence have long suggested that animal and human behavior are short-term oriented and that their discount functions are closer to hyperbolic than exponential (Ainslie 1992; Loewenstein and Prelec 1992). Decades ago, Stroz (1956), Phelps and Pollak (1968) and Laibson (1994) have begun to apply the theory of hyperbolic discounting to consumer's consumption-saving decision problems. Laibson (1996, 1997) further shows that consumers with hyperbolic discounting preferences face undersaving problems, resulting in implications that explain US household saving patterns.

In parallel, the literature in behavioral finance has also suggested that corporate decisions are short-term oriented, and such myopic decisions can result in suboptimal equilibrium (see Stein 1988, 1989; Porter 1992; Bebchuk and Stole 1993; Stein 2003). These theories on corporate short-termism have focused on agency conflicts between corporate managers and stockholders. Corporate managers may underinvest due to pressures from boosting earnings as reflected in stock values. More recently, empirical and survey evidence have demonstrated that short-termism is a prominent feature of corporations

(Asker, Farre-Mensa and Ljungqvist 2015; Budish, Roin and Williams 2015; Poterba and Summers 1995). However, while short-termism has often been argued as a feature of manager's behavior, the theory of hyperbolic discounting has not been formally applied to corporate decisions. This stands in contrast to the large volume of literature applying hyperbolic discounting preferences to consumers' decision. This paper contributes to theories of corporate short-termism by introducing the hyperbolic discounting framework to corporate investments.

This paper shows that a firm with present bias inevitably faces an underinvestment problem. The underinvestment problem arises jointly from (a) the present-biased discounted functions and (b) the supermodularity of the return function. Supermodularity implies that the marginal return of investment in one period is increasing in the investment level of another period. With this property, lower investment in an earlier period raises the incentive to decrease investment in the later period. In the earlier period, present-bias causes the firm to pay more dividend and invest less. Subsequently, this investment cutback behavior in the early period induces the firm to invest less by the supermodularity property. Thus, the low investment in the earlier period (due to its present bias) is underinvestment from the perspective of the later-period, while the low investment in the later period (due to supermodularity) is also underinvestment from the perspective of the earlier firm. Altogether, this implies that the firms' investment decisions are suboptimal in terms of both periods' present values.

Secondly, this paper also provides perspectives on the optimality of dividend taxation and investment subsidies. These two types of policies are commonly introduced to boost investment during periods of recessions.[1]. Our paper suggests that under corporate short-termism, dividend taxes may increase investment, because it has the ability to address the underinvestment problem arising from myopic behavior.[2] Specifically, for this normative ques-

[1]For example, in 2003, the US Congress passed the Jobs and Growth Tax Relief and Reconciliation Act, for which increasing investments was a justification for dividend tax cuts included in the package.

[2]The literature on dividend taxation has debated whether dividend tax cuts exerts a significant effect on investment. Some argue that if corporations finance marginal invest-

tion, we consider dividend taxation in which the government does not collect any net revenue from the firm, as the collected dividend taxes are returned to the firm with lump-sum subsidies. Even with this revenue neutral policy, we show that this type of government intervention improves the firm's present values in all periods. However, taxation in the current period without taxation in other periods inevitably lowers the firm's present value in the current period. Therefore, Pareto-improving investment can only be achieved when the government implements tax policies in all investment periods.

The dividend taxation plan considered increases the relative cost of dividends compared to that of investment. In a static setting, this policy-induced relative cost change would directly decrease dividends and increase investment. However, in a multi-period model, a tax policy in a current period also affects future periods' decisions, and these future decision changes would be in turn be incorporated into the decision of the current period. Therefore, dividend taxation in a period does not necessarily decrease the dividend in that period. We analyze the possible ranges of investment decision changes from each period's tax policy and shows that there exists Pareto-improving dividend tax policies as a combination of these each period's tax policies. We also introduce an investment subsidy policy, where the government provides proportional investment subsidies and collects lump-sum taxes with same amount of investment subsidy. With the same logic as that of the dividend taxation policy, the investment subsidy decreases the cost of investment, and consequently increases the firm's investment level.

This paper shows that policies can generate Pareto-improvement in the firm's value by raising its biased present values. However, O'Donoghue and Rabin (1999) have argued that policy effectiveness should be evaluated with unbiased discounted values. They proposed a long-run "present" value function from a prior perspective, in which the agent weighs all future periods based on unbiased exponential discounting. This long-term perspective cri-

ment through new stocks, dividend tax cuts would increase investment (Chetty and Saez 2004; Poterba and Summers 1995). On the other hand, if marginal investment is financed through retained earnings, then dividend taxes would not affect investment (Auerbach 1979; Bradford 1981).

terion is currently used in the literature for analyzing policy implications.[3] In the corporate finance context, unbiased present value has been interpreted as shareholders' present value, whereas biased present value refers to that of corporate managers. This paper shows that a policy that improves all biased present values is also an improvement based on the long-term perspective unbiased present value.

The main analysis in this paper is based on a three-period model with general return functions. However, we also show that the three-period model can be extended into T-period model (where $T \geq 3$) with Cobb-Douglas return functions. Under this stylized framework, we show that present bias, in general, induces greater decreases in investment the later the period. This phenomenon results from the supermodularity property - lower earlier-period investment decreases the marginal return of later-period investment, providing an additional incentive for the firm to decrease investment in the later period.

The rest of the paper proceeds as follows. In Section 2, we present the set-up of the theoretical framework. In Section 3, we solve for the subgame-perfect Nash equilibrium firm investment levels in our model, define underinvestment, and show that in equilibrium, the firm faces an underinvestment problem. Sections 4 and 5 consider policy solutions to the underinvestment problem. In particular, section 4 shows that a revenue-neutral increase in dividend taxes can overcome the underinvestment problem. Section 5 shows that investment subsidies can also achieve this purpose. Last but not least, section 6 extends the three-period framework to a T-period model with Cobb-Douglas return functions. Section 7 concludes.

2 Three-period model

We first introduce a three-period model of corporate investment decisions under the hyperbolic discounting framework. The firm makes an investment

[3]For policy evaluations based on the "long-run" criterion, see O'Donoghue and Rabin (1999, 2003, 2006), Krusell, et al. (2002), Diamond and Koszegi (2003) and Guo and Krause (2015).

decision in each period to maximize the present value of dividend streams. x_1 and x_2 denote exogenous cash flows in period 1 and 2, respectively. The firm chooses to undertake investments of amounts i_1 and i_2 in period 1 and 2. The return from investments is realized in period 3 and takes on the function $f(i_1, i_2)$. The return function satisfies the Inada conditions and is strictly supermodular (i.e., $\partial^2 f / (\partial i_1 \partial i_2) > 0$). The firm's dividends are denoted as $d_1 = x_1 - i_1$ in period 1, $d_2 = x_2 - i_2$ in period 2, and $d_3 = f(i_1, i_2)$ in period 3. The exogenous cash flows are necessary to avoid negative dividends and do not affect the firm's decisions. We assume that x_1 and x_2 are large enough to avoid the negative dividend.

We apply the popular β, δ functional form in assessing the firm's present values. The present values in periods 1, 2 and 3 are given as

$$PV_1 = d_1 + \beta\delta d_2 + \beta\delta^2 d_3$$

$$PV_2 = d_2 + \beta\delta d_3$$

and

$$PV_2 = d_3$$

With this β, δ functional form, $\beta = 1$ corresponds to exponential discounting, while $\beta \in (0, 1)$ reflects present bias. The present bias is reflected in the model by β, which is an excess discount between the current and the next period.

The use of extra discounting of present values to incorporate short-termism has been established by corporate finance research. This approach is grounded on vast empirical evidence that show corporate discount rates are higher than those implied by efficient markets (King 1972; Poterba and Summers 1995; Miles 1993; Haldane and Davies 2011). Recently, Budish, Roin, and Williams (2015) defined a benchmark discount rate based on the real interest rate and risk factors. Short-termism is defined as an exponential discount rate that is strictly greater than the benchmark discount rate. In our model, the extra discounting applies only to the current and the immediate future period, which deviates from exponential discounting assumption.

We assume that the firm is sophisticated, as defined by knowing how its preferences change over time.[4] The sophisticated firm in period 1 knows how the firm in period 2 makes decisions, given the period-1 decision. Therefore, the equilibrium can be derived in a recursive way. The firm in period 2 chooses i_2 to maximize PV_2, conditional on i_1:

$$\max_{i_2|i_1} \ (x_2 - i_2) + \beta\delta f(i_2, i_1) \tag{1}$$

From the maximization problem (1), we have an implicit function of i_2 in terms of i_1, denoted as $\widehat{i}_2(i_1)$. Even though in most cases closed form of solutions for $\widehat{i}_2(i_1)$ do not exist, we know that $\widehat{i}_2(i_1)$ is a well-defined and strictly increasing function due to the concavity and strict supermodularity of the return function. Thus, with $\widehat{i}_2(i_1)$, the sophisticated firm chooses i_1 to maximize PV_1:

$$\max_{i_1} (x_1 - i_1) + \beta\delta \left(x_2 - \widehat{i}_2(i_1)\right) + \beta\delta^2 f\left(\widehat{i}_2(i_1), i_1\right) \tag{2}$$

3 Underinvestment problem

Having presented the set-up of the model, we analyze how the myopic firm would make suboptimally low levels of investment in equilibrium. In other words, there exists other investment plans that induce higher firm values in all periods. In our paper, the notion of suboptimality of investment arises from the present-biased nature of the firm's discount function. This stands in contrast to existing corporate finance theories of underinvestment and short-termism, the source of which arises from concern over near-term stock prices (Stein 1989). In Stein (1989)'s setting of agency conflicts between corporate mangers and shareholders, there exists a Nash equilibrium solution that leads to underinvestment. In our framework, on the other hand, the relevant game is between the perspective of the firm in the present period and that of

[4] The behavior of present-biased agents can often be different depending on whether they are aware (sophisticated) or unaware (naive) of their self-control problems. O'Donoghue and Rabin (1999, 2015) carefully compares the decision and welfare differences between naive and sophisticated agents.

the future period, and thus the source of underinvestment arises from time-inconsistency. Our framework results in suboptimally low levels of investment in the equilibrium.

Mathematically, the underinvestment problem arises jointly from present-bias ($\beta < 1$) and the supermodularity of the return function ($\partial^2 f / \partial i_1 \partial i_2 > 0$). Supermodularity of the return function means that the marginal return of one period's investment increases in the other period's investment. Consequently, this induces the choice function, $\widehat{i_2}(i_1)$, to be increasing. This implies that the firm will have stronger (weaker) incentive to invest more if investment level is higher (lower) in the past. With present-bias ($\beta < 1$), the firm in period 1 will pay out higher level of dividends and, consequently, lower level of investment from the perspective of period 2. Due to the low level of period-1 investment, period-2 investment is also low because $\widehat{i_2}(i_1)$ is an increasing function. Therefore, low investment levels in both periods result in an underinvestment problem.

To demonstrate the underinvestment problem, we will show the existence of an equilibrium that solves the two maximization problems defined in periods 1 and 2, respectively. Next, we show that marginal increases in both periods' investments from the equilibrium investment level can improve the firm's value in all three periods, which implies that the firm is facing an underinvestment problem. In the following section, we will show that there exists tax and subsidy policies that address this issue by inducing an increase in investment and thus a rise in the firm's value in all periods.

The following proposition shows that there exists an equilibrium(s) from the firm's maximization problem:

Proposition 1 *There exists a subgame perfect Nash equilibrium* $\left(i_1^*, \widehat{i_2}(i_1)\right)$ *such that* $\widehat{i_2}(i_1)$ *solves the period-1 maximization problem, conditional on* i_1*; and* i_1^* *solves the period-2 maximization problem by replacing* i_2 *with* $\widehat{i_2}(i_1)$*.*

Proof. The first-order condition from the maximization problem (1) is

$$-1 + \beta\delta f_2(i_1, i_2) = 0 \tag{3}$$

The second order condition from the maximization problem (1) is

$$\beta\delta f_{22}(i_1, i_2) < 0 \tag{4}$$

By the first and second order conditions, we know that for any value of $i_1 > 0$, there exists an unique $i_2 > 0$ that solves equation (3). We define $\widehat{i_2}(i_1)$, which solves the first order condition in (3), such that

$$-1 + \beta\delta f_2\left(i_1, \widehat{i_2}(i_1)\right) = 0 \tag{5}$$

Implicitly differentiating equation (5) with respect to i_1, we have

$$\beta\delta f_{12}\left(i_1, \widehat{i_2}(i_1)\right) + \beta\delta f_{22}\left(i_1, \widehat{i_2}(i_1)\right)\widehat{i_2'}(i_1) = 0$$

which in turn equivalently is

$$\widehat{i_2'}(i_1) = -\frac{f_{12}\left(i_1, \widehat{i_2}(i_1)\right)}{f_{22}\left(i_1, \widehat{i_2}(i_1)\right)} > 0 \tag{6}$$

The firm maximizes the following in period 1:

$$PV_1 = (x_1 - i_1) + \beta\delta\left(x_2 - \widehat{i_2}(i_1)\right) + \beta\delta^2 f\left(i_1, \widehat{i_2}(i_1)\right)$$

By the Inada conditions and that $\widehat{i_2'}(i_1) > 0$, the optimal solution for i_1^* is neither zero nor infinite. Because $PV_1\left(i_1, \widehat{i_2}(i_1)\right)$ is a smooth function of i_1, by the mean value theorem there is an interior solution i_1^* in which the first order condition is zero and the second order condition is negative.[5] The first order condition is

$$-1 - \beta\delta\widehat{i_2'}(i_1) + \beta\delta^2\left(f_1 + f_2\widehat{i_2'}(i_1)\right) = 0 \tag{7}$$

[5] If $\widehat{i_2}(i_1)$ is linear, the second derivative of $PV_1\left(i_1, \widehat{i_2}(i_1)\right)$ with respect to i_1 is strictly negative globally and, therefore, a unique solution is guaranteed. However, in general, $\widehat{i_2}(i_1)$ is not linear and, in a special case, there can be multiple equilibria. Even though multiple maximum equilibria exists, at the equilibrium the first and second order conditions are well-defined by the mean-value theorem.

The second order condition is

$$-\beta\delta\widehat{i_2''}(i_1) + \beta\delta^2\left(f_{11} + 2f_{12}\widehat{i_2'}(i_1) + f_{22}\left(\widehat{i_2'}(i_1)\right)^2\right) + \beta\delta^2 f_2\widehat{i_2''}(i_1) < 0 \quad (8)$$

■

Next, we will show that the firm's equilibrium decision is suboptimal and the firm experiences the underinvestment problem. We define the underinvestment problem as follows:

Definition 1 *At the equilibrium investment levels* (i_1^*, i_2^*), *the firm faces the underinvestment problem if there exists* $(i_1', i_2') \gg 0$ *such that*

$$i_1' > i_1^*,\ i_2' > i_2^*,$$

$$\overline{PV}_1\left(i_1', i_2'\right) > \overline{PV}_1\left(i_1^*, i_1^*\right),$$

$$\overline{PV}_2\left(i_1', i_2'\right) > \overline{PV}_2\left(i_1^*, i_1^*\right)$$

and

$$\overline{PV}_3\left(i_1', i_2'\right) > \overline{PV}_3\left(i_1^*, i_1^*\right)$$

where

$$\overline{PV}_1\left(i_1, i_2\right) = \left(x_1 - i_1\right) + \beta\delta\left(x_2 - i_2\right) + \beta\delta^2 f\left(i_1, i_2\right),$$

$$\overline{PV}_2\left(i_1, i_2\right) = \left(x_2 - i_2\right) + \beta\delta f\left(i_1, i_2\right)$$

and

$$\overline{PV}_2\left(i_1, i_2\right) = f\left(i_1, i_2\right)$$

Definition 1 states that the firm has the underinvestment problem if there exists another investment plan (i_1', i_2') such that (a) it is strictly higher than the equilibrium investment level (i_1^*, i_2^*) and (b) its associated present values are strictly higher than those of the equilibrium investment decisions. The following proposition shows that based on Definition 1, the firm has an underinvestment problem at the equilibrium:

Proposition 2 *The firm faces an underinvestment problem.*

Proof: The proof of Proposition 2 will be based on the following two lemmas. Lemmas 1 and 2 investigate whether the present value functions $\overline{PV}_1(i_1, i_2)$ and $\overline{PV}_2(i_1, i_2)$ increase or decrease in small variations in (i_1, i_2) at the equilibrium. The present value in period 3, $\overline{PV}_3(i_1, i_2)$, trivially increases in (i_1, i_2).

Lemma 1 *At the equilibrium investment plan* (i_1^*, i_2^*), *we have*

$$\frac{\partial \overline{PV}_1}{\partial i_1} < 0$$

and

$$\frac{\partial \overline{PV}_1}{\partial i_2} > 0$$

Proof of Lemma 1: Taking the partial derivative $\overline{PV}_1$ with respect to i_1 at the equilibrium (i_1^*, i_2^*), we have

$$\frac{\partial \overline{PV}_1}{\partial i_1}|_{(i_1,i_2)=(i_1^*,i_2^*)} = -1 + \beta\delta^2 f_1 \tag{9}$$

From (7) and (9), we have

$$\frac{\partial \overline{PV}_1}{\partial i_1}|_{(i_1,i_2)=(i_1^*,i_2^*)} = -1 + \beta\delta^2 f_1 = \beta\delta\widehat{i'_2}(i_1)\left(1 - \delta f_2\right) \tag{10}$$

From (3) and (10), we have

$$\frac{\partial \overline{PV}_1}{\partial i_1}|_{(i_1,i_2)=(i_1^*,i_2^*)} = \beta\delta^2\widehat{i'_2}(i_1) f_2\left(\beta - 1\right) < 0 \tag{11}$$

Taking the derivative of $\overline{PV}_1$ with respect to i_2 at equilibrium (i_1^*, i_2^*), we have

$$\begin{aligned}\frac{\partial \overline{PV}_1}{\partial i_2}|_{(i_1,i_2)=(i_1^*,i_2^*)} &= -\beta\delta + \beta\delta^2 f_2 \\ &= \beta\delta\left(-1 + \delta f_2\right) \qquad (12)\end{aligned}$$

From (3) and (12), we have

$$\frac{\partial \overline{PV}_1}{\partial i_2}|_{(i_1^*, i_2^*)} = \beta\delta\left(-\beta\delta f_2 + \delta f_2\right) = \beta\delta^2\left(1-\beta\right) f_2 > 0 \tag{13}$$

End of Proof of Lemma 1.

Lemma 2 *At the equilibrium investment plan* (i_1^*, i_2^*)*, we have*

$$\frac{\partial \overline{PV}_2}{\partial i_1} > 0$$

and

$$\frac{\partial \overline{PV}_2}{\partial i_2} = 0$$

Proof of Lemma 2: Taking the partial derivative of $\overline{PV}_2$ with respect to i_2 at equilibrium (i_1^*, i_2^*), we have

$$\frac{\partial \overline{PV}_2}{\partial i_1}|_{(i_1,i_2)=(i_1^*, i_2^*)} = \beta\delta f_1 > 0 \tag{14}$$

The partial derivative of $\overline{PV}_2$ with respect to i_2 is the first order condition (3). Therefore, we have

$$\frac{\partial \overline{PV}_2}{\partial i_2}|_{(i_1,i_2)=(i_1^*, i_2^*)} = 0 \tag{15}$$

End of Proof of Lemma 2.

From Lemmas 1 and 2, in a small open set around the investment equilibrium (i_1^*, i_2^*), there are four different regions, as depicted in Figure 1. Region I is the area where all three present values are higher than those associated with equilibrium investment. Furthermore, in none of the regions is there an overinvestment situation, in which there would exist a lower investment level that leads to Pareto-improving present values in all periods.

End of Proof of Proposition 2.

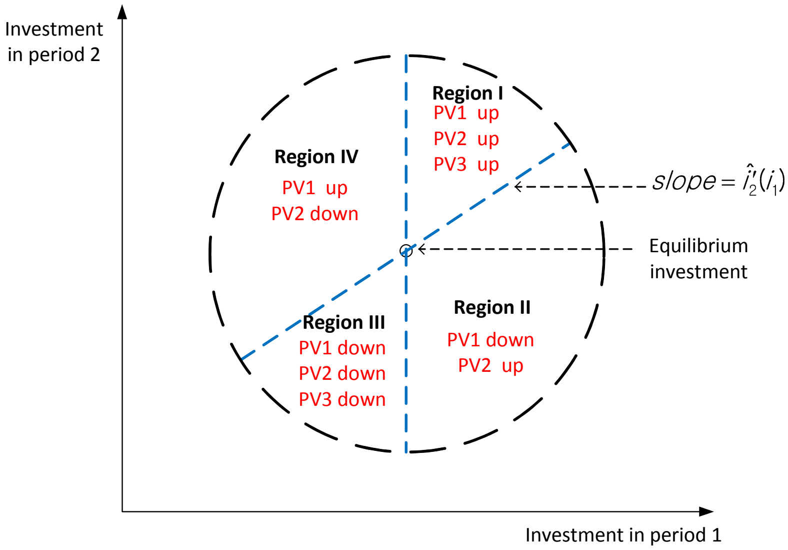

Figure 1: Four regions around the equilibirum investment

From Lemma 2, we know that i_2 must be increasing in order to raise PV_2 at the equilibrium. From Lemma 1, we know that an increase in i_1 decreases PV_1 but increases PV_2. Therefore, i_2 must be increasing sufficiently compared to an increase in i_1 for PV_1 to be increasing. From equation (11) and (13), we have the following:

$$\frac{\partial \overline{PV}_1}{\partial i_1}\Delta i_1 + \frac{\partial \overline{PV}_1}{\partial i_2}\Delta i_2 = di_1\left((\beta-1)\,\beta\delta^2\widehat{i}_2'(i_1)f_2\right) + di_2\beta\delta^2\,(1-\beta)\,f_2 \quad (16)$$

In order for (16) to be strictly positive, $\Delta i_2/\Delta i_1$ needs to satisfy the following inequality:

$$\frac{\Delta i_2}{\Delta i_1} > \widehat{i}_2'(i_1) > 0 \quad (17)$$

Inequality (17) will be used in showing the existence of Pareto-improving tax-policies in the following sections.

As an example in which $f(i_1, i_2) = 20 i_1^{1/4} i_2^{1/6}, \beta = 0.6$ and $\delta = 0.9$, the indifference curves of $\overline{PV}_1(i_1, i_2)$ and $\overline{PV}_2(i_1, i_2)$ are plotted in Figure 2.[6] The

[6]With a Cobb-Douglas return function, there exists a closed form solution for the equilibirum investment (See section 6). In this example, we have $(i_1^*, i_2^*) = (4.69, 3.22)$.

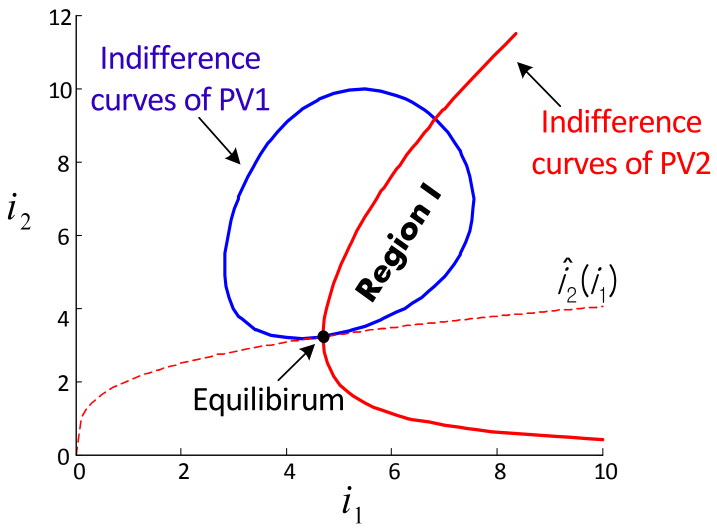

Figure 2: Equilibirum and Pareto-superior region

surrounding region by the two indifference curves are the Pareto-superior region (Region I). The main goal of policies that are introduced in the following sections is to move the equilibrium investment plan into region I in Figure 2.

4 Dividend Taxation

In the previous section, we have shown that myopic corporate decisions result in an underinvestment problem. Now, we move on to policy implications and examine whether outside authorities' intervention can improve the firm's value. For this normative question, we assume that the authority has no exogenous expenditures so that the tax policy is balanced. The collected amount of dividend taxes would be returned to the firm in the form of lump-sum subsidies. We show that even with a revenue-neutral tax policy, the firm's value can be improved.

We examine the effects of proportional dividend taxes on the firm's dividend/investment decisions and present values. Let there be a proportional dividend tax rate τ_t and a lump-sum transfer s_t in period t. The firm's budget sets are

$$(1+\tau_1)\,d_1+i_1=x_1+s_1,$$

$$(1+\tau_2)\,d_2+i_2=x_2+s_2,$$

and

$$d_3=f(i_1,i_2).$$

Since the government has no exogenous expenditure to finance, its budget constraints satisfy $s_t=\tau_t d_t^*$, where d_t^* is the equilibrium dividend in period t. The tax policies τ_1 and τ_2 are fully anticipated and affect both period-1 and period-2 decisions.

For the proof of the existence of Pareto-improving policies, we consider infinitesimal changes of two periods' tax policies at $(\tau_1,\tau_2)=0$ in order to guarantee the existence of an equilibrium. In Proposition 1, we have shown that without tax policies, there exists an equilibrium in which the first and second order conditions are satisfied. The result in Proposition 1 also implies the existence of an equilibrium with $(\tau_1,\tau_2)=0$. However, for any strictly positive tax policy $(\tau_1,\tau_2)>0$, the existence of an equilibrium is not guaranteed, and therefore we need to focus on local analysis in which small changes in tax-policies are considered.

Imposing dividend taxes decreases the marginal cost of investment relative to that of dividends. Because the collected tax is returned as a lump-sum subsidy, an increase in taxes has a substitution effect but not an income effect.[7] The substitution effect, in general, decreases the level of dividend and the level of investment. The following lemma shows that an increase in τ_1 increases both i_1^* and i_2^*.

Lemma 3 *At the equilibrium of* $(\tau_1,\tau_2)=0$*, a (finite) increase in* τ_1 *in-*

[7] It may seem trivial that an increase in the dividend tax in period t causes a decrease in dividend and increase in investment in the same period. However, our context also accounts for the ability of the tax policy in one period to affect the firm's decision in the other period. Because of this intertemporal effect, investment is not necessarily increasing in dividend taxes in the same period. This will be shown for the case of period-2 dividend taxation.

creases the equilibrium investments in both periods, that is

$$0 < \frac{di_1^*}{d\tau_1} < \infty \text{ and } 0 < \frac{di_2^*}{d\tau_1} < \infty. \tag{18}$$

We also have

$$\frac{di_2^*}{d\tau_1} / \frac{di_1^*}{d\tau_1} = \widehat{i_2'}(i_1). \tag{19}$$

Proof. The present value in period 1 is

$$\begin{aligned} PV_1 &= \frac{x_1 - i_1 + s_1}{1+\tau_1} \\ &+ \beta\delta \left(\frac{x_2 - \widehat{i}_2(i_1) + s_2}{1+\tau_2} \right) + \beta\delta^2 f\left(i_1, \widehat{i}_2(i_1)\right) \end{aligned} \tag{20}$$

The first order condition from (20) is

$$-\frac{1}{1+\tau_1} - \beta\delta \frac{\widehat{i_2'}(i_1)}{1+\tau_2} + \beta\delta^2 f_1 + \beta\delta^2 f_2 \widehat{i_2'}(i_1) = 0 \tag{21}$$

Where $(\tau_1, \tau_2) = (0,0)$, the first-order condition in (21) is equivalent to (7) in the proof of Proposition 1. The second order condition from (21) is

$$\begin{aligned} & -\beta\delta \frac{\widehat{i_2''}(i_1)}{1+\tau_2} + \beta\delta^2 f_{11} + 2\beta\delta^2 f_{12} \widehat{i_2'}(i_1) \\ & + \beta\delta^2 f_{22} \left(\widehat{i_2'}(i_1)\right)^2 + \beta\delta^2 f_2 \widehat{i_2''}(i_1) \\ < & \; 0 \end{aligned} \tag{22}$$

Where $(\tau_1, \tau_2) = (0,0)$, the second-order condition in (21) is equivalent to (8) in the proof of Proposition 1. Implicitly differentiating (21) with respect to τ_1, we have

$$\begin{aligned} & \frac{1}{(1+\tau_1)^2} d\tau_1 - \beta\delta \frac{\widehat{i_2''}(i_1)}{1+\tau_2} di_1 \\ & + \beta\delta^2 f_{11} di_1 + \beta\delta^2 f_{12} \widehat{i_2'}(i_1) di_1 + \beta\delta^2 f_{12} \widehat{i_2'}(i_1) di_1 \\ & + \beta\delta^2 f_{22} \left(\widehat{i_2'}(i_1)\right)^2 di_1 + \beta\delta^2 f_2 \widehat{i_2''}(i_1) \\ = & \; 0 \end{aligned} \tag{23}$$

By equation (23) and the second order condition (22), we have

$$0 < \frac{di_1^*}{d\tau_1} < \infty. \tag{24}$$

By (24) and that $\widehat{i}_2'(i_1) > 0$, we have

$$0 < \frac{di_2^*}{d\tau_1} < \infty. \tag{25}$$

and

$$\frac{di_2^*}{d\tau_1} / \frac{di_1^*}{d\tau_1} = \widehat{i}_2'(i_1) \tag{26}$$

■

Lemma 3 shows that period-1 dividend taxation increases investment levels in both periods, and the ratio of the marginal increases of the two investments is equal to $\widehat{i}_2'(i_1)$. The increasing rate $\widehat{i}_2'(i_1)$ implies that if only period-1 dividend taxation is imposed, the Pareto-superior investment plan cannot be achieved (see inequality (17)). Therefore, we also need period-2 dividend taxation. The substitution effect from higher period-2 dividend taxes can increase the choice function $\widehat{i}_2(i_1)$, but does not directly increase the equilibrium period-2 investment, i_2^*. The change of the choice function $\widehat{i}_2(i_1)$ affects period-1 investment choice, and the period-1 investment choice will affect the period-2 investment, i_2^*, through the choice function $\widehat{i}_2(i_1)$. Therefore, whether the two periods' investments increase or decrease from period-2 taxation is not a trivial question. Nevertheless, we can derive the possible range of investment changes by period-2 taxation, which is sufficient to show the existence of Pareto-improving tax policies.[8]

Lemma 4 *At the equilibrium of* $(\tau_1, \tau_2) = 0$*, the following inequality is sat-*

[8] We conjecture that depending on the elasticity of substitution between the two periods' investments, the period-1 investment can increase or decrease from period-2 taxation. In our leading example with Cobb-Dougls production functions, period-1 investment is not affected by period-2 taxation. For the higher elasticity of substitution, increase in period-2 taxation might decrease the period-1 investment because the increased period-1 investment (by period-2 taxation) can substitute the period-1 investment. If the elasticity is smaller than one, the reverse result would be expected. Further studies should be necessary.

isfied:

$$\widehat{i}_2'(i_1)\frac{di_1}{d\tau_2} < \frac{di_2}{d\tau_2}. \tag{27}$$

Proof. The present value in period 2 is

$$PV_2 = \frac{x_2 - i_2 + s_2}{1+\tau_2} + \beta\delta f(i_1, i_2) \tag{28}$$

The first order condition from (28) is

$$-\frac{1}{1+\tau_2} + \beta\delta f_2(i_1, i_2) = 0 \tag{29}$$

Implicitly differentiating (29) with respect to τ_2, we have

$$\frac{d\tau_2}{(1+\tau_2)^2} + \beta\delta f_{22}(i_1, i_2) di_2 = 0$$

and, equivalently,

$$\frac{d\widehat{i}_2(i_1;\tau_2)}{d\tau_2} d\tau_2 = -\frac{1}{(1+\tau_2)^2 \beta\delta f_{22}} > 0. \tag{30}$$

The maximization problem of period-1 present value can be expressed as

$$\max \quad \overline{PV}_1(i_1, i_2)$$

subject to

$$\widehat{i}_2(i_1;\tau_2) = i_2 \tag{31}$$

Taking a total derivative of equation (31) with respect to τ_2, we have

$$\frac{d\widehat{i}_2(i_1;\tau_2)}{d\tau_2} + \widehat{i}_2'(i_1;\tau_2)\frac{di_1}{d\tau_2} = \frac{di_2}{d\tau_2} \tag{32}$$

Because $\frac{d\widehat{i}_2(i_1;\tau_2)}{d\tau_2} > 0$ from (30), we have

$$\widehat{i}_2'(i_1)\frac{di_1}{d\tau_2} < \frac{di_2}{d\tau_2}$$

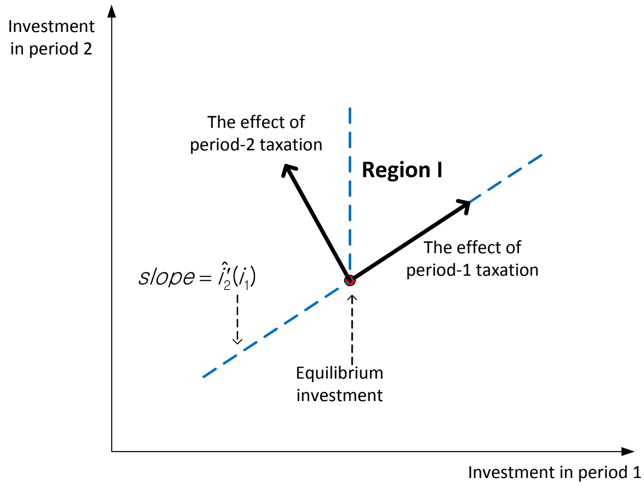

Figure 3: Pareto-improving tax policies

■

Lemma 4 indicates that period-2 taxation induces the equilibrium investment to move above the $\widehat{i_2}(i_1)$ curve (i.e., $\widehat{i_2'}(i_1)\frac{di_1}{d\tau_2} < \frac{di_2}{d\tau_2}$). Inequality (27) does not imply whether period-1 and 2 investments increase or decrease. From Lemmas 3 and 4, the existence of Pareto-improving dividends taxation policies is shown in the following proposition:

Proposition 3 *There exists positive Pareto-improving dividend proportional taxes* $(\tau_1, \tau_2) \gg 0$.

Proof: In the proof, we consider infinitesimal changes in dividend taxes at $(\tau_1, \tau_2) = 0$. Because there exists an equilibrium at $(\tau_1, \tau_2) = 0$, there is also an open set $T \subset \mathbb{R}^2$ such that T includes $(0, 0)$ and that an equilibrium exists for any $(\tau_1, \tau_2) \in T$. Therefore, there still exists a unique equilibrium with infinitesimal variations in (τ_1, τ_2). Lemma 3 indicates that period-1 taxation induces the two-period investment to move along the $\widehat{i_2'}(i_1)$-line in Figure 3 (see (19) in Lemma 3). Inequality (27) in Lemma 4 implies that period-2 taxation induces the investments in both periods to move above the $\widehat{i_2'}(i_1)$-line in Figure 3. Therefore, by combining dividend taxations in both periods, the equilibrium investment can move into the Pareto-superior region

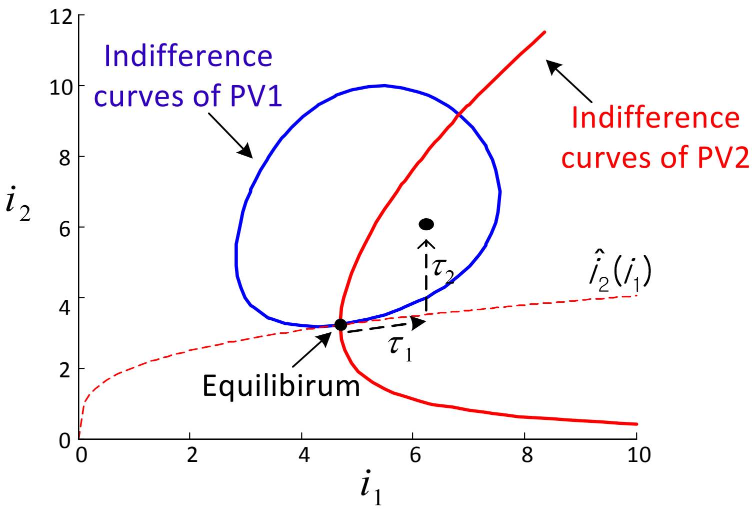

Figure 4: Pareto-improving dividend taxation

(region I). Mathematically, this means that at the equilibrium $(\tau_1, \tau_2) = (0, 0)$, there exist a positive constant a such that

$$0 < \frac{\frac{di_2^*}{d\tau_1} + a\frac{di_2^*}{d\tau_2}}{\frac{di_1^*}{d\tau_1} + a\frac{di_1^*}{d\tau_2}} < \widehat{i}_2'(i_1^*).$$

The end of Proof of Proposition 3.

Figure 4 also describes how dividend taxation policies can Pareto-improve the firm's values. An increase in period-1 tax can move the equilibrium point along the $\widehat{i}_2(i_1)$ curve. Without period-2 taxation, the period-1 taxation cannot improve the period-1 present value (the period-1 present value become even lower along the $\widehat{i}_2(i_1)$ curve). Together with period 1 and 2's tax policies, the equilibrium can move into the Pareto-superior region.

5 Investment subsidy

In this section, we show that investment subsidies can also improve the firm's value in all periods. As in the previous section, we assume that the outside authority adopts revenue-neutral policies. Therefore, lump-sum taxes in the same amount of the investment subsidies will be imposed in the same period. The firm's budget constraints under investment subsidies are

$$d_1 + (1 - \theta_1)\, i_1 = x_1 - t_1,$$

$$d_2 + (1 - \theta_2)\, i_2 = x_2 - t_2,$$

and

$$d_3 = f(i_1, i_2)$$

where s_i and t_i are the proportional subsidy rate and the lump sum tax in period t, respectively. Since the outside authority has no exogenous expenditure to finance, its budget constraints satisfy $t_i = \theta_1 i_i^*$ for $t = 1, 2$, where i_i^* is the the equilibrium investment level in period i.

Following the same logic as the dividend-taxation case in Section 4, an increase in s_1 decreases the cost of investment relative to the cost of dividends. By the substitution effect, an increase in s_1 induces higher equilibrium investment, and therefore, higher present value.

Proposition 4 *There exists positive Pareto-improving investment proportional subsidies* $(\theta_1, \theta_2) >> 0$.

Proof. We do not state the detailed proof of Proposition 4, because the same logic as the proof of Proposition 3 applies. The increase in investment subsidies increases the cost of dividend payout and decreases the cost of investment, which is mathematically equivalent to the case of an increase in dividend taxation. ■

Propositions 3 and 4 show that outside authority's policies can result in Pareto-improvement of the firm's values in all periods. It is also an interesting question whether these Pareto-improving policies also improve the firm "normal" unbiased present values. It may be reasonable that the outside authority evaluates the firm values based on the unbiased exponential discounting rates. This unbiased present value is equivalent to considering the present values using $\beta = 1$.[9] The following Lemma shows that if a policy improves biased present values in all three periods, the policy also improves the unbiased (i.e. $\beta = 1$) present values.[10]

Lemma 5 *Assume that the equilibrium without a policy results in the equilibrium dividends* (d_1^*, d_2^*, d_2^*) *and that with a policy results in* (d_1', d_2', d_2')*. If the present values in all three periods based on* (d_1^*, d_2^*, d_2^*) *is strictly higher than those based on* (d_1^*, d_2^*, d_2^*)*, the unbiased present values with the policy is strictly higher than those without the policy, that is,*

$$d_1' + \delta d_2' + \delta^2 d_3' > d_1^* + \delta d_2^* + \delta^2 d_3^*, \tag{33}$$

and

$$d_2' + \delta d_3' > d_2^* + \delta d_3^*, \tag{34}$$

Proof. We have

$$d_1' + \beta\delta d_2' + \beta\delta^2 d_3' > d_1^* + \beta\delta d_2^* + \beta\delta^2 d_3^*, \tag{35}$$

$$d_2' + \beta\delta d_3' > d_2^* + \beta\delta d_3^*, \tag{36}$$

and

$$d_3' > d_3^*. \tag{37}$$

Inequality (36) can be expressed as

$$d_2' + \delta d_3' - (1-\beta)\,\delta d_3' > d_2^* + \delta d_3^* - (1-\beta)\,\delta d_3^*. \tag{38}$$

Multiplying $(1-\beta)$ in inequality (37) and adding it to inequality (38), we have

$$d_2' + \delta d_3' > d_2^* + \delta d_3^*. \tag{39}$$

[9] With quasi-hyperbolic discouting time preferences, O'Donoghue and Rabin (1999) first proposed a different welfare criterion that considers an agency's perspective in a fictitious period 0. This "long-run" perspective welfare criterion is equivalent to considering the intertemporal utility of period 1 using $\beta = 1$.

[10] In any finite model with quasi-hyperbolic discounting time preferences, Kang (2015) showed that if a policy leads to Pareto-improvement in all existing periods, it also does so in the fictitious period.

Inequality (35) can be expressed as

$$\begin{aligned} & d_1' + \delta d_2' + \delta^2 d_3' - (1-\beta)\,\delta\,(d_2' + \delta d_3') \\ > \; & d_1^* + \delta d_2^* + \delta^2 d_3^* - (1-\beta)\,\delta\,(d_2^* + \delta d_3^*)\,. \end{aligned} \tag{40}$$

Multiplying $(1-\beta)\,\delta$ in inequality (39) and adding it to inequality (40), we have

$$d_1' + \delta d_2' + \delta^2 d_3' > d_1^* + \delta d_2^* + \delta^2 d_3^*.$$

■

Lemma 5 implies that the Pareto-improving policies in Propositions 3 and 4 are also raising the firm's unbiased present values. Therefore, we have the following Corollary

Corollary 1 *There exist positive proportional taxes* (τ_1, τ_2) *that improves the firm's unbiased present values. There exist positive Pareto-improving investment proportional subsidies* (θ_1, θ_2)*,*

Proof. Directly from Propositions 3 and 4, and Lemma 5. ■

6 Multi-period Case with Cobb-Douglas Return Function

We have shown that a firm with hyperbolic preferences faces the underinvestment problem in a three-period model. In this section, we introduce a multi-period model with Cobb-Douglas return function under quasi-hyperbolic discounted preferences.[11] We will show that in this multi-period Cobb-Douglas setting, the firm also faces an underinvestment problem if $\beta < 1$ (i.e. present bias). In Section 3, we have shown that the three-period model can be reduced into a two-period maximization problem by plugging a choice function

[11] In a three-period model, there is no mathematical distinction between hyperbolic discounting and quasi-hyperbolic discouting. However, over more than three periods, these two discounting concepts are different. Psychologists first proposed hyperbolic discounting but economic theorists more frequently use quasi-hyperbolic discounting time preferences, mainly due to computational convenience.

into the original three-period model. In the same way, a four-period model can be reduced into a three-period model with a choice function of the last-period investment, and so on. In this section, we show that a multi-period model with Cobb-Douglas return function can be solved in a recursive way, in which we consecutively reduce the T period model into $T-1$, $T-2$, and up to a 3-period model.

This recursive way is feasible with a Cobb-Douglas return function because the derived return function in a reduced model is also a Cobb-Douglas function: any T-period model (where $T \geq 3$) can be reduced into a 3-period model with another Cobb-Douglas return function. This "preservation" property of the Cobb-Douglas return function is not satisfied under other return functions, such as non-Cobb-Douglas CES functions. Using the main result of this section, we present examples showing how investment levels are changing over time for different values of β. Finally, we will show that with Cobb-Douglas return functions, present bias combined with supermodularity decrease late-period investments more than early-period investments.

Consider the Cobb-Douglas return function in a T-period model. The return function is given by

$$f\left(i_1, i_2 ...., i_{T-1}\right) = z \prod_{s=1}^{a_s} i_s^{a_s}$$

where

$$z > 0,\ a_j > 0 \text{ for all } j \in \{1, ..., t-1\}, \text{ and } \sum_{j=1}^{T-1} a_j < 1.$$

With quasi-hyperbolic discounting, proposed by Laibson (1997), the present value of period t is defined as

$$\begin{aligned} PV_t &= (x_t - i_t) + \beta \sum_{s=1}^{T-1-t} \delta^s \left(x_{t+s} - i_{t+s}\right) + \beta \delta^{T-t} f\left(i_1, i_2 ...., i_{T-1}\right) \\ \text{where } 1 &\leq t \leq T-1 \end{aligned}$$

and

$$PV_t = f\left(i_1, i_2 ...., i_{T-1}\right) \quad \text{where } t = T,$$

where x_t is an exogenous cash flow in period t. We assume that the cash flow in each period is large enough to avoid negative dividends. If $\beta = 1$, then (b, d)-present values are simply exponential discounting. However, $b < 1$ implies present-biased present values. Thus, the firm gives more relative weight to the period-t dividend in period t than it did in any period prior to t.

With this multi-period Cobb-Douglas return function, there exists an equilibrium and the equilibrium possesses an underinvestment problem. The following Proposition addresses this issue:

Proposition 5 *For any finite period $T \geq 3$, there exists a unique equilibrium under the Cobb-Douglas return function. At the equilibrium, the firm faces an underinvestment problem.*

Proof. To simplify the notation, we drop the cash flow x_t in the maximization problem. Because the exogenous cash flows are eliminated in the first order conditions, they do not affect the firm's investment decisions. In period $T-1$ and for any given $(i_1, i_2, ..., i_{T-2})$, the firm solves the following maximization problem:

$$\max_{i_{T-1}|\{i_s\}_{s=1}^{T-2}} -i_{T-1} + \beta\delta f\left(i_1, i_2...., i_{T-1}\right) \tag{41}$$

From maximization problem (41), we can derive a choice function $\widehat{i}_{T-1} : \mathbb{R}_{++}^{T-2} \to \mathbb{R}_{++}$ such that

$$\widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2}) = \{\beta\delta a_{T-1} f\left(i_1, ..., i_{T-2}, 1\right)\}^{\frac{1}{1-a_{T-1}}} \tag{42}$$

The maximization problem in period $T-2$ is given as

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} - \beta\delta\widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2}) + \beta\delta^2 f\left(i_1, i_2...., \widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2})\right), \tag{43}$$

which is equivalent in turn, to

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} + \beta\delta\left\{-\widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2}) + \delta f\left(i_1, i_2...., \widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2})\right)\right\} \tag{44}$$

Using (42), the expression inside $\{\cdot\}$ in (44) can be expressed as

$$\begin{aligned}
& \{-\beta\delta a_{T-1} f(i_1, ..., i_{T-2}, 1)\}^{\frac{1}{1-a_{T-1}}} + \delta f\left(i_1, i_2...., \widehat{i}_{T-1}(\{i_s\}_{s=1}^{T-2})\right) \\
= & \ (\beta\delta a_{T-1})^{\frac{1}{1-a_{T-1}}} f(i_1, ..., i_{T-2}, 1)^{\frac{1}{1-a_{T-1}}} \\
& + \delta f(i_1, ..., i_{T-2}, 1) \{\beta\delta a_{T-1} f(i_1, ..., i_{T-2}, 1)\}^{\frac{a_{T-1}}{1-a_{T-1}}} \\
= & \left(\delta (\beta\delta a_{T-1})^{\frac{a_{T-1}}{1-a_{T-1}}} - (\beta\delta a_{T-1})^{\frac{1}{1-a_{T-1}}}\right) f(i_1, ..., i_{T-2}, 1)^{\frac{1}{1-a_{T-1}}} \qquad (45) \\
= & \ (\beta\delta a_{T-1})^{\frac{1}{1-a_{T-1}}} \left(\frac{1-\beta a_{T-1}}{\beta a_{T-1}}\right) f(i_1, ..., i_{T-2}, 1)^{\frac{1}{1-a_{T-1}}}
\end{aligned}$$

Defining a function $f^{(T-2)} : \mathbb{R}_{++}^{T-2} \to \mathbb{R}_{++}$:

$$f^{(T-2)}\left(\{i_s\}_{s=1}^{T-2}\right) = (\beta\delta a_{T-1})^{\frac{1}{1-a_{T-1}}} \left(\frac{1-\beta a_{T-1}}{\beta a_{T-1}}\right) f(i_1, ..., i_{T-2}, 1)^{\frac{1}{1-a_{T-1}}}, \tag{46}$$

which is also a Cobb-Douglas function that is strictly increasing and strictly concave. With the function in (46), the maximization problem in (43) can be expressed as

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} + \beta\delta f^{(T-2)}\left(\{i_s\}_{s=1}^{T-2}\right), \tag{47}$$

which is defined in a same way as the maximization problem of (41). Therefore, we have the choice function $\widehat{i}_{T-2}$, given as

$$\widehat{i}_{T-2} = \left\{\beta\delta a^{(T-2)} f^{(T-2)}(i_1, ..., i_{T-3}, 1)\right\}^{\frac{1}{1-a^{(T-2)}}} \tag{48}$$

where

$$a^{(k)} = \frac{\partial f^{(k)}\left(\{i_s\}_{s=1}^{T-2}\right)}{\partial i_k}|_{\{i_1,..,i_k\}=\{1,...,1\}} \tag{49}$$

In a recursive way, we define a function $f^{(k)} : \mathbb{R}_{++}^{k} \to \mathbb{R}_{++}$ such that

$$f^{(k)} = \left(\beta\delta a^{(k+1)}\right)^{\frac{1}{1-a^{(k+1)}}} \left(\frac{1-\beta a^{(k+1)}}{\beta a^{(k+1)}}\right) f^{(k+1)}(i_1, ..., i_k, 1)^{\frac{1}{1-a^{(k+1)}}}, \tag{50}$$

Then, the maximization problem in period k is expressed as

$$\max_{i_k|\{i_s\}_{s=1}^{k-1}} -i_k + \beta\delta f^{(k)}\left(\{i_s\}_{s=1}^{k}\right). \tag{51}$$

Since the function $f^{(k)}$ is strictly concave and strictly increasing, the maximization problem in period k has a unique solution for any given $\{i_s\}_{s=1}^{k-1}$. In period 1, we have a unique solution, i_1^*, from the maximization problem (51) with $k = 1$. This unique solution is used to get a unique solution i_2^* from the maximization problem with $k = 2$. Repeating this process, we have a unique investment equilibrium, $\{i_s^*\}_{s=1}^{T-1}$.

Next, we move on to the underinvestment issue. From Proposition 2, we know that in period $T-2$, for any given $(i_1, i_2, ..., i_{T-3})$, the equilibrium investment

$$(i_{T-1}^*, i_{T-2}^*) = \left(\widehat{i}_{T-1}\left(\{i_s\}_{s=1}^{T-3}, \widehat{i}_{T-2}(\{i_s\}_{s=1}^{T-3}\})\right), \widehat{i}_{T-2}\left(\{i_s\}_{s=1}^{T-3}\}\right)\right) \tag{52}$$

is underinvestment in the sense that there is another investment plan (i'_{T-1}, i'_{T-2}), which (a) is strictly greater than the equilibrium investment level and (b) the corresponding present values in periods $T-2$, $T-1$ and T are strictly higher than those in the equilibrium. In the maximization problem in period $T-4$ and $T-3$, replacing the variable i_{T-1} with a choice function $\widehat{i}_{T-1}\left(\{i_s\}_{s=1}^{T-2}\}\right)$, we have a "derived" 3-period model with Cobb-Douglas function. Specifically, in period $T-3$, the maximization problem is

$$\max_{i_{T-2}|\{i_s\}_{s=1}^{T-3}} -i_{T-2} - \beta\delta\left\{\widehat{i}_{T-1} + \delta f(i_1, ..., i_{T-2}, \widehat{i}_{T-1})\right\} \tag{53}$$

and in period $T-4$, the maximization problem is

$$\max_{i_{T-3}|\{i_s\}_{s=1}^{T-4}} -i_{T-3} - \beta\delta\widehat{i}_{T-2} - \beta\delta^2\left\{\widehat{i}_{T-1} + \delta f(i_1, ..., \widehat{i}_{T-2}, \widehat{i}_{T-1})\right\} \tag{54}$$

where $\widehat{i}_{T-2} = \widehat{i}_{T-2}\left(\{i_s\}_{s=1}^{T-3}\}\right)$ and $\widehat{i}_{T-1} = \widehat{i}_{T-1}\left(\{i_s\}_{s=1}^{T-3}, \widehat{i}_{T-2}(\{i_s\}_{s=1}^{T-3}\})\right)$. We have shown that the term $\left\{\widehat{i}_{T-1} + \delta f(i_1, ..., \widehat{i}_{T-2}, \widehat{i}_{T-1})\right\}$ is well-defined, strictly increasing, and strictly concave (see (45)). Therefore, by Proposition

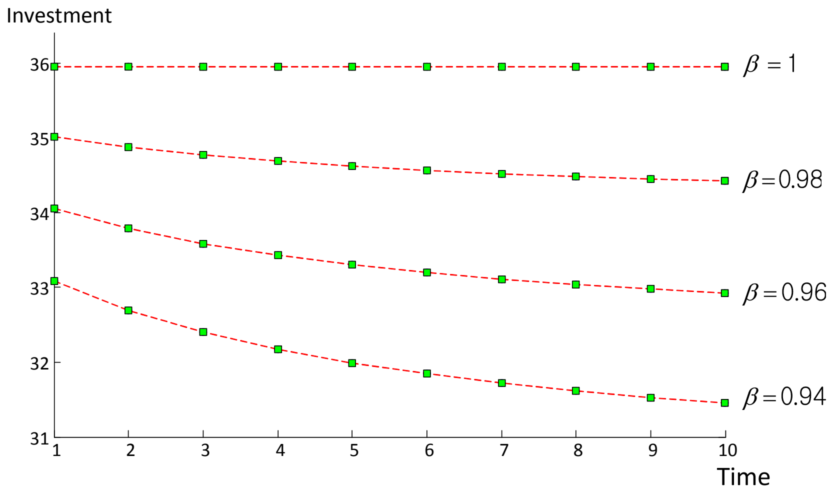

Figure 5: Investment over time

2, for any given $(i_1, ..., i_{T-4})$ the equilibrium investment plan

$$(i^*_{T-2}, i^*_{T-3}) = \left(\widehat{i}_{T-2}\left(\{i_s\}_{s=1}^{T-3}, \widehat{i}_{T-3}\left(\{i_s\}_{s=1}^{T-4}\right)\right), \widehat{i}_{T-3}\left(\{i_s\}_{s=1}^{T-4}\right)\right) \tag{55}$$

is underinvestment. From (52) and (55), we know that any given $(i_1, ..., i_{T-4})$, the equilibrium investment $(i^*_{T-3}, i^*_{T-2}, i^*_{T-1})$ is underinvestment in the sense that there is another investment plan $(i'_{T-3}, i'_{T-2}, i'_{T-1})$ that induces higher present values in period $T, T-1, T-2$ and $T-3$. Repeating this process, we can show that the equilibrium investment $(i^*_1, i^*_2..., i^*_{T-1})$ is underinvestment.
■

Proposition 5 shows that for any Cobb-Douglas return function and for any finite number of periods, there exists an equilibrium investment plan and the equilibrium decisions possess an underinvestment problem if $\beta < 1$. The equilibrium investment plan can be analytically and recursively solved from equations (48–49) in the proof of Proposition 5. In Figure 5, we show the investment plans across different values of β in a model with 11 periods. The example is based on the return function: $f(i_1, ..., i_{10}) = 100 \prod_{s=1}^{10} i_s^{a_s}$ where $a_1 = 0.1$ and $a_{t-1} = a_t\delta$ for $t = \{2, ..., 10\}$. The discount rate is $\delta = 0.9$.

Figure 5 clearly shows that investment is decreasing (constant) in time if

$\beta < 1$ ($\beta = 1$). In general, the existence of present bias decreases all period's investment levels. However, due to the supermodularity property of the return function (i.e. marginal product of one period's investment is positively related to the levels of other periods' investments), present bias uneven impacts investments across different time period. Therefore, the low investment levels from the earlier periods will decrease the marginal return of later period investment, and thus the firm in the later-period will have an incentive to decrease investment further. The combination of present bias and supermodularity causes the later-period investment to be even lower compared to earlier-period investments. On the other hand, the supermodularity property affects the earlier-period investments differently. The firm in the earlier period knows that the low investment in the current period will decease future investments and also knows that low future investment will decrease the marginal product of current investment. Therefore, the supermodularity property provides the early-period firm an incentive to decrease investment less intensively than in the later period. The following proposition shows that present-bias disproportionately affects the later-period investment as compared to the earlier-period investment.

Proposition 6 *For any finite period $T \geq 3$ with a Cobb-Douglas return function satisfying $a_t = a_{t-1}\delta$ for all $t \in \{2, ..., T\}$, investment level is strictly decreasing (constant) over time if $\beta < 1$ ($\beta = 1$).*

Proof. To simplify the notation, we drop the cash flow x_t in the maximization problem in the same way as the proof of Proposition 5. The choice function $\widehat{i}_{T-1}(i_1, ..., i_{T-1})$ is strictly increasing in i_{T-1} from (42). We have shown that if the return function is supermodular, the choice function is strictly increasing in a three-period model (See (6) in Section 2). Given $(i_1, ..., i_{T-2})$, the maximization problem in period $T-1$ is

$$\max_{i_{T-1}|\{i_s\}_{s=1}^{T-2}} -i_{T-1} + \beta\delta f\left(i_1, i_2...., i_{T-1}\right),$$

and its first-order condition is

$$-1 + \beta\delta \frac{\partial f}{\partial i_{T-1}} = 0. \tag{56}$$

Given $(i_1, ..., i_{T-3})$, the maximization problem in period $T-2$ is

$$\max_{i_{T-2} | \{i_s\}_{s=1}^{T-3}} -i_{T-2} - \beta\delta \widehat{i}_{T-1}(\cdot) + \beta\delta^2 f\left(i_1, i_2 ...., \widehat{i}_{T-1}(\cdot)\right),$$

and its first-order condition is

$$-1 - \beta\delta \frac{\partial \widehat{i}_{T-1}}{\partial i_{T-2}} + \beta\delta^2 \frac{\partial f}{\partial i_{T-2}} + \beta\delta^2 \frac{\partial f}{\partial i_{T-1}} \frac{\partial \widehat{i}_{T-1}}{\partial i_{T-2}} = 0. \tag{57}$$

From equations (56) and (57), we have

$$\delta \left(\frac{\partial f}{\partial i_{T-2}}\right) / \frac{\partial f}{\partial i_{T-1}} = 1 - \delta \frac{\partial \widehat{i}_{T-1}}{\partial i_{T-2}} (1 - \beta). \tag{58}$$

Because we have $a_t = a_{t-1}\delta$, the left term in equation (58) is

$$\delta \left(\frac{\partial f}{\partial i_{T-2}}\right) / \frac{\partial f}{\partial i_{T-1}} = \delta \frac{i_{T-1}^{a_{T-1}} a_{T-2} i_{T-2}^{a_{T-2}-1}}{a_{T-1} i_{T-1}^{a_{T-1}-1} i_{T-2}^{a_{T-2}}} = \delta \frac{a_{T-2}}{a_{T-1}} \frac{i_{T-1}}{i_{T-2}} = \frac{i_{T-1}}{i_{T-2}}. \tag{59}$$

From (58) and (59), we have

$$\frac{i_{T-1}}{i_{T-2}} = 1 - \delta \frac{\partial i_{T-1}}{\partial i_{T-2}} (1 - \beta), \tag{60}$$

which implies that for any given $(i_1, ..., i_{T-3})$, if $\beta < 1$ $(\beta = 1)$, we have $i_{T-1} > i_{T-2}$ $(i_{T-1} = i_{T-2})$. In the same recursive way as in the proof of Proposition 5, plugging the choice function $\widehat{i}_{T-1}(\cdot)$ into the maximization problems in periods $T-4$ and $T-3$, we have the following equation

$$\frac{i_{T-2}}{i_{T-3}} = 1 - \delta \frac{\partial \widehat{i}_{T-2}}{\partial i_{T-3}} (1 - \beta), \tag{61}$$

which also implies that for any given $(i_1, ..., i_{T-4})$, if $\beta < 1$ $(\beta = 1)$, we have $i_{T-2} > i_{T-3}$ $(i_{T-2} = i_{T-3})$. Repeating this recursive analysis, we know that the equilibrium investment i_t is strictly decreasing (constant) in t if $\beta < 1$ ($\beta = 1$). ■

In Proposition 6, in order to show the investment decisions across time, we need a benchmark case. We consider a special case of $\beta = 1$, where investment is constant across time in the model. The condition for the constant investment stream is $\delta \left(\frac{\partial f}{\partial i_{t-1}} \right) / \frac{\partial f}{\partial i_t} = 1$, which is the condition $a_t = a_{t-1}\delta$ with a Cobb-Douglas return function. Then, where $\beta = 1$, the ratio of marginal product to marginal cost is identical for all periods, and consequently equilibrium investment levels are identical across time. If $\beta < 1$, investment is decreasing over time in the special case, which is shown in Proposition 6.

However, the key equation in Proposition 6, equation 58, not only applies to the Cobb-Douglas return function, but also any return function in a three-period model. Equation (58) in the three-period model is

$$\delta \left(\frac{\partial f}{\partial i_1} \right) / \frac{\partial f}{\partial i_2} = 1 - \delta \widehat{i_2'}(i_1) (1 - \beta) . \tag{62}$$

In equation (62), we know that the two properties of $\widehat{i_2'}(i_1) > 0$ (due to the supermodularity) and $\beta < 1$ (due to present bias) induce the marginal product of period-2 investment to be relatively higher than that of period-1 investment. The higher marginal product in period-2 equilibrium investment implies disproportionately lower level of period-2 equilibrium investment due to diminishing marginal product of the return function. Therefore, we can conclude that supermodularity in general amplifies present-bias-induced underinvestment problems for later-stage investment decisions.

7 Conclusion

We construct a theoretical framework that incorporates hyperbolic discounting preferences into corporate investment decisions. In doing so, we rigorously establish the linkage between short-termism and underinvestment. In our three-period framework, the firm with present-bias makes investment decisions that result in suboptimally low levels of investment, as defined by the existence of a higher-level investment plan that improves all periods' present value of dividends. We then conduct two policy analyses that can overcome this underinvestment problem: dividend taxation and investment subsidies. We show that revenue-neutral dividend taxes and investment subsidies can correct the market distortions imposed by present biases. Finally, in a finite multi-period extension of the model, we demonstrate that the underinvestment induced by hyperbolic discounting preferences is uneven across time. The firm's underinvestment problem is more severe as time elapses. The analysis in this paper provides theoretical underpinnings for arguments in the policy arena that advocate corporate taxation as a method for addressing corporate short-termism.

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