Optimal contract for asset trades: Collateralizing or selling?

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Abstract

We develop a dynamic model to study the conditions under which assets are sold or used as collateral. When the borrower has an incentive to falsify the assets' quality, they cannot be sold directly but can be used as collateral via over-collateralization. Secured loan contracts can also be optimal by reducing the lender's incentive to acquire costly information about the assets' future value. However, under secured loan contracts, the borrower may default opportunistically. Thus, an asset sale can be optimal under some conditions. The model also provides the theoretic explanation on the negative correlation between interest rates and haircuts.

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1 Introduction

When an asset holder has liquidity needs but cannot issue unsecured debt, he/she can raise funds with the asset in two different ways: the agent can sell the asset directly, or he/she can use the asset as collateral to raise funds for liquidity. Existing studies on secured loan contracts often rely on asymmetric valuation of collateral assets. More precisely, a borrower values the asset more than a lender (see Lacker 2001, Ferraris and Watanabe 2008, Antinolfi et al. 2014, Monnet and Narajabad 2012, Zhang 2014, Tomura 2016, Williamson forthcoming). However, the intrinsic value of financial assets does not necessarily depend on the identity of the asset holder, and trillions of dollars of financial assets, such as government bonds and asset-backed securities (ABS), are traded daily in repo and security lending markets as collateral (see Baklanova, Copeland, and McCaughrin 2015 and Gorton and Metrick 2012). At the same time, financial assets that are traded in these markets are also traded immediately on a spot market without any repurchase agreement.

Furthermore, the literature has paid relatively little attention to the properties of secured loan contracts, such as haircuts and over-collateralization. Here, a haircut is the percentage difference between the collateral value and the loan size, and we state that a loan is over-collateralized if the collateral value is higher than the value of the repayment on the loan. For example, suppose an agent borrows \$80 at an interest rate of 10% and posts an asset as collateral whose market value is \$100. Then, the haircut is $20\%[= (100 - 80)/100]$ and the over-collateralization ratio is $12\%[= (100-88)/100]$. In particular, over-collateralization is puzzling because standard theory suggests that the risk is priced into the interest rate, and as long as the repayment value is the same as the collateral value, the lender is fully insured.[1] However, over-collateralization is a common practice in secured loan markets. For example, data from the Korea Securities Depository shows that during the period from 2010 to 2017, 98.5% of overnight repo transactions between financial institutions in Korea were over-collateralized. More importantly, in models that do not explicitly consider over-collateralization, interest rates and haircuts are essentially the same indicator, even though the denominators are different. Thus, in those types of models, interest rates and haircuts respond in the same way to all sources of risk, so they have limited ability to explain the

[1] Collateralized debt contracts can be also under-collateralized, i.e., they can have a negative over-collateralization ratio, but collateralized debt contracts typically involve over-collateralization.

recent empirical finding that shows substitution effects between interest rates and haircuts (see Baklanova et al. 2017).

In this paper, we attempt to make progress in filling the gap between theory and practice in the real world. In particular, we answer the following questions: Under what conditions do economic agents use assets as collateral or sell assets to obtain needed liquidity when the intrinsic value of the assets is the same across all agents? When are secured loan contracts over-collateralized and for what reason? What are the determinants of the interest rate, haircut, and over-collateralization ratio of secured loan contracts?

For this purpose, we construct an exchange model of optimal trading arrangement. In the model, there are two periods and two risk neutral agents. Because of a misallocation of resources, it is socially efficient that the borrower consumes the lender's goods in the first period and compensates the lender in the second period. However, the borrower cannot issue unsecured debt due to limited commitment problem. Instead, the borrower owns perfectly divisible Lucas trees that can be used as a medium of exchange in the first period, and the trees give stochastic dividends at the end of the second period. In principle, the borrower can use the trees in the first period as a medium of exchanges in two different ways. First, the borrower can sell the trees to the lender in exchange for consumption goods (asset sale). Second, he/she can pledge the trees as collateral to borrow consumption goods from the lender (secured loan contract). In a standard exchange model, such as the models based on Lagos and Wright (2005), these two types of trading arrangements generally generate the same economic result (see Lagos 2010 and Rocheteau 2011).

One of the key assumptions in the model is that in the first period, the lender can acquire private information about the dividend state of the trees at a cost before trading with the borrower. For example, an agent can purchase analytic reports that evaluate a company's future prospects more precisely than common perceptions before purchasing an equity share of the company. The borrower, on the other hand, receives the information about the dividend state of the trees at the beginning of the second period with some probability, which resonates with the idea of "learning by holding" proposed by Plantin (2009), whereby the owner of an asset can obtain private information about the future cash flow of the asset by holding the asset. This implies that if the borrower makes a secured loan contract in the first period and learns that the value of repayment on the loan is higher than the value of the collateral trees, the borrower will default on the loan opportunistically.

The other important assumption is that the borrower can create counterfeit trees at a cost and use them as a medium of exchange in the first period. This incentive problem generates a fraud incentive constraint that can restrict the extent to which the trees can facilitate the exchange process, similar to Li, Rocheteau, and Weill (2012). In the model, counterfeiting represents the moral hazard problem of misrepresenting the quality of assets, such as fraudulent asset appraisal in financial markets. These types of fraudulent practices were prevalent in the U.S. market before the 2008 financial crisis; in particular, securitized bonds such as mortgage backed securities (MBS) were prey for quality manipulation because the complicated structure of their securitization made it challenging to pierce the veil of MBS (see Keys et al. 2010, Gorton and Metrick 2012, and Piskorski, Seru, and Witkin, 2015). Checking the future value of an asset and verifying the authenticity of the asset clearly require different skills and information. Thus, we assume that the information about the dividend state that the lender can acquire does not reveal the authenticity of trees.

We first restrict our attention to secured loan contracts that provide the highest payoff to the borrower among secured loan contracts. The terms of direct sales, can then, be easily obtained by imposing the exogenous condition that the borrower always defaults on loans and thus cedes collateral trees with certainty into the terms of the secured loan contract. The type of secured loan contracts can be categorized into two groups depending on whether the lender acquires costly information about the dividend state or not. First, under information insensitive contracts (IIS), the lender does not acquire costly information. Second, information sensitive contracts (IS), on the other hand, induce the lender to acquire costly information about the dividend state. The interest rates of IIS loan contracts represent compensation for the opportunistic default and may include informational rent that deters the lender from information acquisition. Under IS loan contracts, the lender accepts the borrower's offer only if the dividend state is good, so the borrower cannot default opportunistically. Instead, the borrower must compensate the lender for the information acquisition cost in the form of a positive interest rate. It is clearly more likely that IIS loan contracts will dominate IS loan contracts as the information acquisition cost rises, and vice versa.

In addition to the information acquisition incentive, the fraud incentive also matters for the terms of contracts. If the borrower pledges counterfeit trees, he/she can avoid making repayment without losing genuine trees, so the benefit of fraud increases with the value of repayment given the quantity of collateral trees. Therefore, when the fraud incentive problem matters, the borrower can mitigate the fraud incentive by reducing the size of the repayment below the value of the collateral asset, i.e., over-collateralizing the loan. In this case, interest rates and haircuts can respond in a different way to a shock on a particular set of parameters that represent the economic environment and the asset's properties, thus providing a theoretical explanation for the finding of a negative correlation between interest rates and haircuts (see Baklanova et al. 2017). Furthermore, if the trees are safe, i.e., the dividend state is good with sufficiently high probability, then the interest rates on secured loans can be negative, which sheds light on the episodic experiences of negative interest rates in repo markets.

In the model, secured loan contracts can be optimal for two reasons. First, when the fraud incentive to cheat on the authenticity of the trees is severe due to a low counterfeiting cost, the borrower cannot sell the trees directly to the lender because of the threat of fraud. However, by over-collateralizing the loan, the borrower can mitigate the fraud incentive, so the trees can be pledged as collateral. Thus, a secured loan contract is optimal. Second, under a secured loan contract, the lender seizes the collateral trees only if the borrower defaults on the loan, rather than with certainty, as is the case with asset sales. Therefore, the lender has less incentive to obtain costly information about the dividend state compared to the direct sales of trees. Thus, if the incentive constraint that deters the lender from information acquisition under information insensitive (IIS) contracts binds, then a secured loan contract can be better than direct sales of trees because it reduces the lender's incentive to obtain costly information. On the other hand, if information sensitive (IS) loan contracts are the best among secured loan contracts due to a sufficiently low information acquisition cost, then direct sales also induce information acquisition, and, in that case, secured loan contracts are equivalent to the direct sales of trees.

However, when the fraud and information acquisition incentives do not exist due to high costs of counterfeiting and information acquisition, secured loan contracts only give the borrower an option to default opportunistically whenever possible. The borrower clearly must pay a cost to obtain such an option to make the lender accept the offer. Thus, when both information acquisition incentive and fraud incentive do not exist, secured loan contracts can be suboptimal because of opportunistic default, and the direct sale of the trees emerges as the optimal contract.

Literature review A related stream of literature examines the optimal contract for asset trades. Tomura (2016) shows that an agent uses assets as collateral instead of selling them to overcome the hold-up problem in over-the-counter markets. However, he assumed that a lender values collateral assets strictly less than any other agents, including the borrower, to generate the hold-up problem. Monnet and Narajabad (2012) examine a dynamic search model in which agents' valuations of the asset follow a stochastic process, and the authors show that agents prefer renting assets, which is interpreted as a repo contract, to purchasing them when they face substantial uncertainty about the exogenous private value of holding the asset due to the hold-up problem in a bilateral trade. Parlatore (forthcoming) goes a step further and shows that when an asset is not perfectly liquid and investment opportunities are persistent, borrowing firms value the asset more than lenders, even though the intrinsic value of the asset is independent of the identity of the asset holders because firms can raise funds to invest in profitable projects, so secured loans are optimal. Dang, Gorton, and Holmström (2012) and Madison (2017) investigate how (potential) information asymmetries regarding the future value of assets make secured loan contracts optimal for asset trades.

This study differs from the previous literature in two respects. First, while most of the previous literature focuses on the optimality of secured loan contracts, we also explicitly analyze economic conditions in which an asset sale strictly dominates secured loan contracts. Tomura (2016) shows that if the lender cannot dispose of collateral assets when default occurs, a spot trade can be optimal. However, this result is an outcome of the assumption that the lender values the assets less than other agents. On the other hand, we show that asset sales can be optimal even when there is no intrinsic difference in the asset valuations between agents due to the informational friction.[2]

Second, we comprehensively analyze the determinants of interest rates, haircuts, and the ratio of over-collateralization. A few studies have examined the interest rates and haircuts of secured loan contracts, but to the best of our knowledge, no systematic analysis has conducted on over-collateralization, even though it is common in practice. Dang, Gorton, and Holmström (2012) and Tomura (2016) derived over-collateralization in their models although

[2] In Monnet and Narajabad (2012), agents optimally choose how much to sell and rent their assets when their private valuation of the assets is low. The authors show that agents sell more assets rather than renting them when they face little uncertainty regarding the future use of the assets, but renting becomes more prevalent as the level of uncertainty increases. However, their result demonstrates the quantity dominance of selling or renting rather than the dominance of a particular trading arrangement.

they do not discuss it explicitly in their studies. However, over-collateralization occurs in Tomura (2016) because of asymmetric valuations of collateral assets between the borrower and the lender, and it exists in Dang, Gorton, and Holmström (2012) because of an assumption that the interest rates on loans are zero. More precisely, in Dang, Gorton, and Holmström (2012), the lender may have to sell collateral assets at a lower price when default occurs. The borrower compensates the lender for taking such a risk by over-collateralizing the loan, which means a positive haircut given the zero interest rate, so the lender can obtain some profit when default occurs. Without the zero interest rate assumption, over–collateralization does not occur in their model. In contrast, over-collateralization occurs in our model to circumvent the moral hazard problem in financial markets, and more importantly, we analyze interest rates, haircuts, and over-collateralization together in one framework. By doing so, our model provides theoretical explanations for the possibility of negative interest rates and the substitution effects between interest rates and haircuts that were documented in a recent empirical study when loans are over-collateralized.

Our paper is also related to the literature that studies the effects of information in asset exchange models. Andolfatto and Martin (2013) and Andolfatto, et al. (2014) show that the nondisclosure of information about the future value of an asset can enhance the asset's role as a medium of exchange, and generally improve the social welfare. Gorton and Ordoñez (2014) construct a dynamic model to investigate the macro dynamics of a lack of information production by private agents about collateral assets and show that a small aggregate shock can cause a credit crunch. While the main focus of these studies is on the implications of information about assets' liquidity, we are interested in the optimal contract for asset trades. Furthermore, in our model, information production can facilitate exchanges by eliminating opportunistic default, in contrast to previous studies.

The macroeconomic implications of the threat of fraudulent practices in financial markets have been explored recently in a series of papers. Li, Rocheteau, and Weill (2012) introduce costly counterfeiting technology into Lagos and Wright (2005) framework to investigate how the counterfeiting incentive affects assets' liquidity, prices, and welfare. In particular, they show that the cost of producing counterfeit assets generates an upper bound on the quantity of assets that can be traded in the OTC market. Williamson (forthcoming) examines how monetary policy affects faking incentives in the banking sector and mortgage market. Kang (2018) extends Williamson (forthcoming) to analyze the effects of central bank's private asset purchases when commercial banks have an incentive to misrepresent the quality of private assets, and the author shows that the central bank should purchase private assets only if the bank's moral hazard problem is sufficiently severe. In all of these models, the threat of fraud restricts the volume of asset trading, and collateralized debt contracts and asset sales are equivalent. Our approach goes beyond this earlier literature on fraudulent practices in financial markets by investigating how the threat of fraud affects the optimal contract for asset trades. More precisely, we show that the threat of fraud can generate over-collateralization instead of an upper bound on the quantity of assets that can be traded, which makes collateralized debt contracts strictly better than asset sales.

The rest of the paper is organized as follows. Section 2 presents the environment of the model and section 3 solves the bargaining problem to find the optimal contract. Section 4 concludes the paper. All omitted proofs are provided in the Appendix.

2 The Model Economy

We consider an exchange economy that consists of two agents-a borrower (b) and a lender (l)- and two periods, $t \in \{0, 1\}$. The utility of each agent is

$$U^b = mc_{b0} - l_{b0} + c_{b1}$$
$$U^l = c_{l0} - l_{l0} + c_{l1},$$

where m is the marginal value of consumption of the borrower at $t = 0$, c_{it} and l_{it} are the utility from the consumption of goods and the disutility from the labor of agent $i \in \{b, l\}$ in period t. We assume that $m > 1$, which can be interpreted as the borrower has liquidity needs in period $t = 0$. There is a single non-durable consumption good in each period, and its endowment process is as follows. The lender is endowed with a substantial amount of consumption good e in period $t = 0$ and receives nothing in period $t = 1$. On the other hand, the borrower does not receive any consumption goods in period $t = 0$ and is endowed with e units of consumption goods in period $t = 1$ with probability $1 - \alpha \in [0, 1]$. In addition to consumption goods, the borrower is also endowed with a units of the divisible Lucas tree that can be interpreted as equity, bonds, asset-backed securities, or real assets such as housing at $t = 0$. One unit of a tree yields y units of consumption goods at the end of period $t = 1$ with probability $\sigma \in [\frac{m-1}{m}, 1]$ (good state), and it yields nothing with complement probability (bad state).[3] Let $\overline{y} = \sigma y$ denote the expected dividend of each tree. We assume that $e > ya$ so that there are enough consumption goods that can be traded with trees. The dividend state is realized at the end of period $t = 1$.

Given the utility functions and endowment process, there are gains from trading in period $t = 0$. However, because of a lack of commitment, unsecured credit is not feasible because the borrower would always default on his/her obligation. Therefore, trees are necessary as a medium of exchange for a trade to occur in period $t = 0$. The borrower can finance the liquidity needs in period $t = 0$ using trees in one of two ways. On the one hand, the borrower can sell a' units of trees to the lender in exchange for q units of consumption goods (an asset sale). On the other hand, the borrower can borrow consumption goods from the lender by pledging trees as collateral (secured loan contract). A secured loan consists of three terms, (q, p, a'): It period $t = 0$, the borrower receives q units of consumption goods from the lender, and promises to repay p units of consumption goods in period $t = 1$. Thus, the interest rate on the collateralized debt is $r = \frac{p-q}{q}$. At the same time, the borrower posts a' units of trees as collateral in period $t = 0$. Thus, if the borrower fails to make repayment, then the lender seizes the collateral trees. This transaction is akin to a repo contract, in which the borrower sells a' units of trees with a repurchase agreement that the borrower can repurchase the collateral trees with p units of consumption goods in period $t = 1$.[4] When bargaining in period $t = 0$, we assume that the borrower makes a take-it-or-leave-it offer to the lender.

In period $t = 0$, the borrower and the lender value the trees equally, so there is no intrinsic difference in the tree valuations between agents. In this environment, if there are no other frictions, then the borrower could purchase $\overline{y}a$ units of goods from the lender in exchange for a units of trees in period $t = 0$, given risk neutral preferences of each agent. At the same time, the borrower can borrow $\overline{y}a$ units of goods in period $t = 0$ by pledging a units of

[3] If $\sigma \in \left[0, \frac{m-1}{m}\right)$, then it is possible that the optimal secured loan contract switches from IS-1 (or IS-2) to IIS-1 (or IIS-4) in the proposition 1 later. Otherwise, extending the range of σ does not admit any important insight. For example, it does not affect any properties of secured loan contracts and the economic mechanism that affects the type of optimal trading arrangement for tree trades.

[4] In practice, there are some differences in the legal properties between collateralized debt contracts and repo contracts. In particular, a repo contract is exempt from the automatic stay, so a repo lender can liquidate the collateral assets immediately once a borrower defaults. Antinolfi et al. (2014) examine how this exemption from an automatic stay affects the financial market and what is the optimal bankruptcy policy.

trees as collateral, and he/she promises to repay $\overline{y}a$ units of goods in period $t = 1$ if he/she receives the endowment. These two types of trading arrangements are incentive compatible, and socially efficient. More importantly, the borrower is indifferent between selling trees and pledging them as collateral. Therefore, absent any additional frictions, secured loans and asset sales are equivalent, and in the following, we describe the specific frictions that could break this equivalence result in the economy.

Costly Information Acquisition In reality, an economic agent may want to acquire more information about the future value of an asset than its expected value before purchasing the asset. For example, an agent may obtain analytic reports about the financial statements of a company and its future prospects that might provide more precise information about the value of an equity share of the company even though there is a common perception about the expected value of the company's equity before making the investment decision about that company. Similarly, when an agent considers purchasing houses, he/she may gather more detailed information about the living environment and direction of government policy that could affect housing values in the neighborhood. It is certainly costly to obtain these types of information. One may have to purchase research reports from analysts or exert one's own effort and time to obtain detailed information. To capture this practice, we assume that the lender can acquire private information about the dividend state of trees before trading with the borrower in period $t = 0$. To obtain this information, the borrower must incur a fixed cost of $\gamma > 0$ in terms of labor in period $t = 0$.

Defaults on a secured loan Under secured loan contracts, a borrower may default on the loan for two reasons. First, the borrower is not able to repay the loan because he/she does not have enough resources. This is captured by parameter α in the model. With probability α, the borrower does not receive any consumption goods in period $t = 1$, so he/she cannot make a repayment on the loan. Second, the borrower may default on the loan even though he/she has sufficient resources because it is profitable to him. Specifically, the borrower will compare the value of the collateral to the value of the avoided repayment, and will default optimally when the latter is higher than the former. We introduce this type of default into the model in a simple way. As Plantin (2009) argues, the owner of an asset may obtain private information about the future cash flow of the asset by holding the asset, which is dubbed as "learning by holding". Thus, we assume that the borrower receives private information about the dividend state of trees at the beginning of period $t = 1$ with probability $\eta \in [0, 1]$, before settling any debts. Therefore, if the borrower learns that trees yield nothing, then he/she will default on the loan in period $t = 1$. Note that the borrower receives information about the dividend state after trading with the lender, not before making an offer to the lender. Thus, the borrower does not have private information when he/she makes an offer to the lender in period $t = 0$.[5] We make this timing assumption to avoid signaling problems and to make the analysis as simple as possible but without compromising the economic intuition.

Fraud in financial affairs Misrepresenting the quality of financial assets has been prevalent throughout history. Counterfeiting money has a long history that goes back to the clipping of coins in ancient Rome, and the economic effects of counterfeiting money have been an important topic in monetary economics (Williamson 2002, Nosal and Wallace 2007, Li and Rocheteau 2011, Kang 2017). However, money is not the only asset that has been a victim of fraud. For example, the complicated securitization process of asset backed securities (ABS) has made it difficult to pierce the veil of ABS, and this lack of recognizability problem in ABS markets made ABS the target of fraudsters. Fraudulent asset appraisals of ABS with rating deficiencies and false documentation concerning the underlying assets were common before the financial crisis, and fraudulent activities the in financial market were criticized as one of key factors in the financial crisis of 2008 (see Barnett 2012, Gourinchas and Jeanne 2012, and the Financial Crisis Inquiry Report 2011).[6] Furthermore, fraudulent practices are not restricted to financial assets. Mortgage markets are also susceptible to mortgage fraud, such as misrepresenting the quality of collateral houses. One example is property flipping, which involves the purchase and subsequent resale of property at an artificially inflated price that enables the purchaser to obtain a greater loan and then de-

[5] In contrast to our study, Hopenhayn and Werner (1996), Velde, Weber, and Wright (1999), and Rocheteau (2011) examine how informational asymmetries regarding the future value of an asset can affect the asset's role in transactions and its liquidity when the owner of the asset has private information about the future cash flow of the asset before making an offer.

[6] Robert Lucas, in an interview with the Wall Street Journal (Sep. 24, 2011), also emphasized this fraudulent practice in the financial market as a key factor in the financial crisis. He argued that "Instead, the shock came because complex mortgage-related securities minted by Wall Street and "certified as safe" by rating agencies had become part of the effective liquidity supply of the system. All of a sudden, a whole bunch of this stuff turns out to be crap. It is the financial aspect that was instrumental in the meltdown of '08."

fault. Although no central repositor collects data on all mortgage fraud, Suspicious Activity Reports by financial institutions indicate that the size of mortgage fraud is not negligible.

We introduce the incentive problem of misrepresenting asset quality in the following way. The borrower can produce fraudulent trees that give no dividend at a proportional cost of k units of labor and can trade them with the lender, similar to the faking technology of Williamson (forthcoming). We assume that the lender can obtain information about the dividend state of trees at a cost, but this information does not reveal the authenticity of the trees. Checking the authenticity of assets would clearly require different information and skills in reality.

To simplify the signaling problem, we assume that the borrower decides whether to produce fake trees after making an offer to the lender but before the lender decides whether to accept the offer. Therefore, the borrower makes a fraud decision given the terms of the trade, which disciplines the lender's belief. However, the analysis and results do not hinge on this timing assumption. Although the borrower makes the faking decision first before making an offer, we obtain the same results as long as we use the reordering invariance equilibrium concept proposed by In and Wright (2017) to refine the equilibria as demonstrated by Li, Rocheteau, and Weill (2012) and Kang (2017). In equilibrium, the borrower will not produce fake trees, and the fraud possibility generates an incentive constraint (e.g., see Li, Rocheteau, and Weill, 2012 and Williamson, forthcoming). Figure 1 summarizes the sequence of events in the economy.

3 Optimal contract for tree trades in period $t = 0$

In this section, we study the terms of trade that the borrower offers to the lender in period $t = 0$. As discussed in the previous section, the borrower can obtain consumption goods from the lender either by selling trees directly (asset sale) or collateralizing trees (secured loan contract). However, in the model, we can interpret the tree sale as a special case of a secured loan with $\alpha = 1$, because in that case, the borrower cannot repay the loan, and the lender will seize the collateral trees with certainty. Therefore, we focus on a secured loan contract from now on, and compare secured loan contracts and tree sales later.

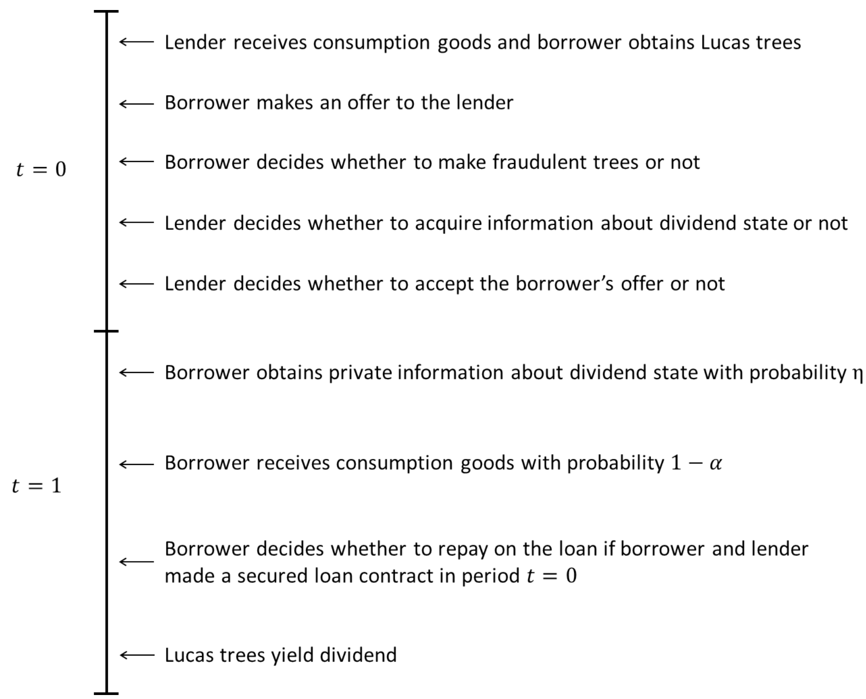

Figure 1: Timeline of borrower's and lender's decision

3.1 Secured loans

Under a secured loan, when the borrower defaults on the loan, the lender seizes the collateral trees. In that circumstance, the lender's final payoff from the secured loan contract depends on the dividend state of the trees. Thus, the lender has an incentive to acquire information about the dividend state. However, if the lender believes that default will not occur or will occur with very low probability, he/she will not acquire costly information about the dividend state of the trees. On the other hand, if the lender has a belief that default could occur with high probability, he/she may want to check the dividend state of the trees before trading with the borrower. Thus, the lender's payoff from the loan contract (q, p, a') depends on whether he/she acquires costly information about the dividend state.

First, when the lender does not acquire the information, lender's expected payoff from a secured loan contract (q, p, a') is

$$\pi_{IIS} = -q + (1 - \alpha)[1 - (1 - \sigma)\eta]p + \alpha\overline{y}a'. \tag{1}$$

Once the lender accepts the borrowers offer, the lender transfers q units of consumption goods to the borrower in period 0. Then, the lender receives either p units of consumption goods from the borrower or seizes a' units of collateral trees in period 1. With probability α, the borrower does not receive any endowments in period 1, so he/she cannot repay p. As a consequence, the lender seizes the collateral trees, which give $\overline{y}a'$ units of dividend in expectation. On the other hand, with probability $1-\alpha$, the borrower receives consumption goods in period $t=1$, so he/she is able to make repayment p on the loan. However, if the borrower learns that the trees will give no dividend, then he/she will default on the loan even if the borrower is able to repay the debt. The term $(1-\alpha)(1-\sigma)\eta p$ in equation (1) represents the lender's loss from this opportunistic default because the lender does not receive any dividend from the collateral trees in that case.

Second, when the lender acquires information about the dividend state of the trees, he/she will accept the borrower's offer only if the dividend state is good. Therefore, the lender's expected payoff is

$$\pi_{IS} = \sigma\left[-q + (1-\alpha)p + \alpha y a'\right] - \gamma. \tag{2}$$

Note that the borrower cannot default opportunistically in this case, because the lender only accepts the borrower's offer when the dividend state is good. However, the lender must incur γ units of labor cost to acquire private information.

Given the terms of trade (q, p, a'), if $\pi_{IIS} \geq \pi_{IS}$, then the lender will accept the borrower's offer without information acquisition. On the other hand, if $\pi_{IIS} < \pi_{IS}$, then the lender acquires costly information about the dividend state first and makes an acceptance decision based on the information about the dividend state. Following the language of Dang, Gorton, and Holström (2012), we state that a secured loan contract is information insensitive (IIS) if it does not trigger information acquisition by the lender, and otherwise, the contract is information sensitive (IS).

Similarly, the borrower's surplus from trading depends on the lender's information acquisition decision. Specifically, an IIS contract ensures a trade with certainty as long as the lender's participation constraint is satisfied, but the borrower can conduct a trade only if the dividend state is good under an IS contract. Thus, our strategy of finding the optimal secured loan contract is as follows. First, we solve the borrower's problem under each type of loan contract - IIS and IS. Then, we compare the borrower's payoff under each type, and choose a secured loan contract that yields the highest surplus to the borrower.

3.1.1 Information insensitive (IIS) loan contracts

We first start with an IIS secured loan contract under which the lender does not acquire costly information about the dividend state. Note that information about the dividend state is not produced under IIS loan contracts, so the expected value of collateral trees when a trade occurs is $\overline{y}a'$, given terms of contract (q, p, a'). Therefore, a haircut θ, defined as the difference between the collateral value and the size of granted loan, is $\theta = \frac{\overline{y}a' - q}{\overline{y}a'}$.

Next, as we explained above, the borrower could make fraudulent trees at the proportional cost of k. If the borrower pledges fraudulent trees as collateral, then he/she can default in the next period without losing genuine trees. Thus, the borrower can save $(1-\alpha)[1-\eta(1-\sigma)]p + \alpha\overline{y}a'$ units of consumption goods from fraud, but he/she has to pay the ka' units of labor to produce a' units of fraudulent trees. Given the terms of trade (q, p, a'), the payoff from fraud should not be higher than the fraud cost. Otherwise, the lender would not accept the borrower's offer. This generates the fraud incentive constraint in the borrower's problem below. Then, the borrower's maximized value under IIS loan contracts, V_{IIS}, is given by

$$V_{IIS} = \underset{q,p,a'}{Max} \{mq - (1-\alpha)[1-(1-\sigma)\eta]p - \alpha\overline{y}a' + \overline{y}a\} \tag{3}$$

subject to

$$-q + (1-\alpha)[1-(1-\sigma)\eta]p + \alpha\overline{y}a' \geq 0 \tag{4}$$

$$-(1-\sigma)q + (1-\eta)(1-\alpha)(1-\sigma)p + \gamma \geq 0 \tag{5}$$

$$ka' - (1-\alpha)[1-(1-\sigma)\eta]p - \alpha\overline{y}a' \geq 0 \tag{6}$$

$$\overline{y}a' - p \geq 0 \tag{7}$$

$$a - a' \geq 0 \tag{8}$$

$$q, p, a' \geq 0 \tag{9}$$

The objective function (3) consists of the borrower's surplus from trade, $mq - (1-\alpha)[1-(1-\sigma)\eta]p - \alpha\overline{y}a'$, and the expected value of the tree holdings $\overline{y}a$. The inequality

(4) is the lender's participation constraint without information acquisition. (5) is the no-information acquisition constraint that deters the lender from the production of information about the dividend state, which means that the lender's payoff with information acquisition (2) should not be higher than the payoff without information acquisition (1). Next, (6) is the fraud incentive constraint that prevents the borrower from producing counterfeit trees. The inequality (7) implies that the value of the avoided repayment is not higher than the expected value of the collateral trees, so the borrower has an incentive to make the repayment unless he/she receives private information stating that the dividend state is bad. Here, if $p < \overline{y}a'$, then we call the secured loan is over-collateralized and define $\frac{\overline{y}a'-p}{\overline{y}a'}$ as the over-collateralization ratio. Note that if $p = \overline{y}a'$, then the positive interest rate on the secured loan $r = \frac{p-q}{q}$ manifests itself as the positive haircut $\theta = \frac{\overline{y}a'-q}{\overline{y}a'}$, although the denominator is different. Thus, we focus on the analysis of the interest rate when $p = \overline{y}a'$ and analyze the interest rate and haircut separately only if the secured loan is over-collateralized, i.e., $p < \overline{y}a'$. Finally, (8) and (9) are the feasibility constraints.

The solution to problem (3) depends on the fraud cost k, the information acquisition cost γ, and the probability that the borrower does not receive endowments α because of their effects on the no-information acquisition constraint (5) and the fraud incentive constraint (6). Depending on the fraud cost k, the solution can be divided into three groups, and we analyze each case in a separate lemma that describes the terms of IIS loan contracts in the following.

Lemma 1 *Suppose* $[1-\eta(1-\alpha)(1-\sigma)]\overline{y} \leq k$ *and define* $\gamma^*_{IIS} \equiv [\alpha + (1-\alpha)\eta\sigma](1-\sigma)\overline{y}a$ *and* $\alpha^* \equiv \frac{(1-\eta)(m-1)-\eta\sigma}{1+(1-\eta)(m-1)-\eta\sigma}$.

1. *[IIS-1] If* $\gamma^*_{IIS} \leq \gamma$, *then* $q = [1-\eta(1-\alpha)(1-\sigma)]\overline{y}a$, $p = \overline{y}a$, $a' = a$, *and* $V_{IIS} = [m-(m-1)\eta(1-\alpha)(1-\sigma)]\overline{y}a$.
2. *[IIS-2] If* $\gamma < \gamma^*_{IIS}$ *and* $\alpha \leq \alpha^*$, *then* $q = (1-\alpha)(1-\eta)\overline{y}a + \frac{\gamma}{1-\sigma}$, $p = \overline{y}a$, $a' = a$, *and* $V_{IIS} = m\left[(1-\alpha)(1-\eta)\overline{y}a + \frac{\gamma}{1-\sigma}\right] + (1-\alpha)(1-\sigma)\eta\overline{y}a$.
3. *[IIS-3] If* $\gamma < \gamma^*_{IIS}$ *and* $\alpha > \alpha^*$, *then* $q = \frac{[1-(1-\alpha)(1-\sigma)\eta]\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)}$, $p = \frac{\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)}$, $a' = \frac{\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)\overline{y}}$, *and* $V_{IIS} = \frac{(m-1)[1-(1-\alpha)(1-\sigma)\eta]\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)} + \overline{y}a$.

When $[1-\eta(1-\alpha)(1-\sigma)]\overline{y} \leq k$, the fraud cost is sufficiently high, so the fraud incentive constraint (6) does not bind. In this case, the constraint (7) binds, i.e., $p = \overline{y}a'$. The type and terms of the secured loan contract depend on the parameters that affect the lender's incentive for information acquisition: the information acquisition cost γ and the probability α that the borrower does not receive endowments in period $t = 1$. Here, the effect of γ on the lender's incentive is straightforward. As γ increases, the lender has less incentive to acquire the information. Next, α is the probability that the borrower is not able to make a repayment on the loan in the period $t = 1$. Thus, as α increases, it becomes more likely that the lender will seize the collateral trees, so the lender has a higher incentive to obtain the information about the future value of the trees.

If $\gamma \geq \gamma^*_{IIS}$, then the information acquisition cost is high enough that the lender has no incentive to acquire information, so the no-information acquisition constraint (5) does not bind (IIS-1 type). Under a collateralized debt contract, the borrower can default on the loan in a profitable way when he/she receives private information about the dividend state at the beginning of period $t = 1$. Because the lender knows the possibility of opportunistic default, the borrower has to compensate the lender for taking such a risk. This is given by the positive interest rate on the loan $r = \frac{(1-\alpha)(1-\sigma)\eta}{1-(1-\alpha)(1-\sigma)\eta}$. Note that if $\eta = 0$ so the borrower cannot default opportunistically, then the interest rate is zero. As the probability that the borrower does not receive endowment α or the probability of good dividend state σ increases, there is less chance of opportunistic default. Thus, the interest rate falls as α or σ rises.

However, when the information acquisition cost γ is low, such that $\gamma < \gamma^*_{IIS}$, the no-information acquisition constraint (5) starts to bind. The specific type of IIS loan contract depends on the probability α that the borrower is not endowed in period $t = 1$ because of its effects on the information acquisition incentive of the lender: As α increases, the lender has more incentive to acquire information.

First, if $\alpha \leq \alpha^*$ (IIS-2 type), then the borrower will be able to repay the loan with relatively high probability, so the lender has relatively less incentive to obtain information about the dividend state even though the constraint (5) binds. In this case, the borrower posts all trees as collateral, i.e., $a' = a$, and reduces the lender's incentive to produce information by reducing the loan size q in (5). Note, that a decrease of q relaxes the lender's participation constraint (4) more than the no-information acquisition constraint (5). Since (4) binds under the IIS-1 contract, the borrower must provide a positive surplus to the lender under the IIS-2 loan contract in order to discourage the lender from information acquisition with a reduction of the loan size q. This is the informational rent to the lender, and the rent rises as the information acquisition incentive increases. Hence, the interest rate,

$r = \frac{[\eta(1-\alpha)+\alpha]\overline{y}a - \frac{\gamma}{1-\sigma}}{(1-\alpha)(1-\eta)\overline{y}a + \frac{\gamma}{1-\sigma}}$, contains the compensation for the risk of opportunistic default and the informational rent. Note that even if $\eta = 0$, the interest rate on the IIS-2 loan contract is still positive because of the informational rent.

The effects of η on the interest rate is the same with the IIS-1 case above: a higher η means a higher probability of opportunistic default, so the borrower must provide a higher interest rate, similar to the IIS-1 loan contract. However, because of the binding no-information acquisition constraint (5), the probability of exogenous default α has two opposing effects on the interest rate. First, as explained above, the borrower has less chance to default on the loan in an opportunistic way as α increases, which pushes down the interest rate. Second, as α increases, the probability that the lender ends up holding the ownership of the collateral trees increases. Therefore, the lender has more incentive to acquire costly information about the future value of the trees, which raises the informational rent and pushes up the interest rate. In the IIS-2 case, the second effect dominates the first one, so the interest rate increases with α. By the same reasoning, as the information acquisition cost γ decreases, the lender's information acquisition incentive rises, so the interest rate increases. Finally, because the lender is concerned about the bad dividend state of the trees, the information acquisition incentive decreases with σ. Thus, the interest rate on the loan falls as σ rises. In the extreme case, if the trees always yield dividends, i.e., $\sigma = 1$, then the information has no bites, and (5) does never bind.

Second, if $\alpha > \alpha^*$ (IIS-3 type), the borrower defaults on the loan with relatively high probability. Thus, the lender has a high incentive to acquire information about the dividend state. In this circumstance, it is too costly to discourage the lender from information acquisition by providing the informational rent to the lender. Instead, the borrower posts only a fraction of the trees as collateral, i.e., $a' < a$, and reduces q and p appropriately, to reduce the lender's information acquisition incentive. Given all other things equal, as the quantity of collateral trees decreases, the value of information about the future value of the trees decreases. Thus, the lender has less incentive to acquire the information given the fixed cost of information acquisition. Here, the quantity of collateral trees, $a' = \frac{\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)}$, increases with γ and decreases with α because of their effects on the information acquisition incentive. Because the borrower does not give the informational rent to the lender in this case, the interest rate on the collateral loan, $r = \frac{(1-\alpha)(1-\sigma)\eta}{1-(1-\alpha)(1-\sigma)\eta}$, is the same as the IIS-1 case.

Lemma 2 *Suppose* $\alpha\overline{y} \leq k < [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$*, and define* $\gamma^{**}_{IIS} \equiv \frac{[\sigma\eta k+(1-\eta)\alpha\overline{y}](1-\sigma)a}{1-\eta(1-\sigma)}$ *and* $\alpha^{**} \equiv \frac{k[(m-1)(1-\eta)-\eta\sigma]}{m(1-\eta)\overline{y}}$.

1. *[IIS-4] If* $\gamma^{**}_{IIS} \leq \gamma$*, then* $q = ka$, $p = \frac{(k-\alpha\overline{y})a}{(1-\alpha)[1-\eta(1-\sigma)]}$, $a' = a$*, and* $V_{IIS} = (m-1)ka + \overline{y}a$.
2. *[IIS-5] If* $\gamma < \gamma^{**}_{IIS}$ *and* $\alpha \leq \alpha^{**}$*, then* $q = \frac{(1-\eta)(k-\alpha\overline{y})a}{1-\eta(1-\sigma)} + \frac{\gamma}{1-\sigma}$, $p = \frac{(k-\alpha\overline{y})a}{(1-\alpha)[1-\eta(1-\sigma)]}$, $a' = a$*, and* $V_{IIS} = m\left\{\frac{(1-\eta)(k-\alpha\overline{y})a}{1-\eta(1-\alpha)} + \frac{\gamma}{1-\sigma}\right\} - ka + \overline{y}a$.
3. *[IIS-6] If* $\gamma < \gamma^{**}_{IIS}$ *and* $\alpha > \alpha^{**}$*, then* $q = \frac{[1-\eta(1-\sigma)]\gamma k}{[\eta k+\alpha(1-\eta)y](1-\sigma)\sigma}$, $p = \frac{(k-\alpha\overline{y})\gamma}{[\eta k+\alpha(1-\eta)y](1-\alpha)(1-\sigma)\sigma}$, $a' = \frac{[1-\eta(1-\sigma)]\gamma}{[\eta k+\alpha(1-\eta)y](1-\sigma)\sigma}$*, and* $V_{IIS} = \frac{(m-1)[1-\eta(1-\sigma)]\gamma k}{[\eta k+\alpha(1-\eta)y](1-\sigma)\sigma} + \overline{y}a$.

Lemma 3 *Suppose* $k < \alpha\overline{y}$*. Then, an IIS secured loan contract is not feasible.*

Now consider the case where $k < [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$, so the borrower has an incentive to use fraudulent trees as a medium of exchange, and hence the lender may be reluctant to trade with the borrower because of the threat of fraud. In this environment, the borrower can mitigate the fraud incentive problem in the following way. As explained above, the borrower can save $(1-\alpha)[1-\eta(1-\sigma)]p + \alpha\overline{y}a'$ units of consumption goods in expectation by transferring fraudulent trees to the lender. Thus, given the quantity of collateral trees, a', the benefit from fraud decreases as p falls, while the cost of producing fraudulent mortgages does not change. Therefore, the borrower can give a signal about the authenticity of the collateral trees to the lender by over-collateralizing the loan, i.e., $p < \overline{y}a'$.

However, when $k < \alpha\overline{y}$, the fraud incentive problem is so severe that the borrower cannot circumvent it and cannot issue IIS secured debt. On the other hand, if $\alpha\overline{y} \leq k < [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$, the fraud incentive is not too high, and, hence, an IIS secured loan contract is feasible with the binding fraud incentive constraint (6) and over-collateralization. The ratio of over-collateralization, $\frac{\overline{y}a'-p}{\overline{y}a'}$, is the same for all types (IIS-4 to IIS-6), as $\frac{[1-\eta(1-\alpha)(1-\sigma)]\overline{y}-k}{(1-\alpha)[1-\eta(1-\sigma)]\overline{y}}$, and it increases (decreases) with α and σ (k and η), because of their effects on the fraud incentive: the payoff from fraud, $(1-\alpha)[1-\eta(1-\sigma)]p + \alpha\overline{y}a'$, increases with α and σ, decreases with η, and the borrower has less incentive to commit fraud as the fraud cost k increases.

Although the binding fraud incentive constraint (6) generates the over-collateralization, the effects of γ, α, and a' on the lender's information acquisition incentive and the structure of IIS loan contracts, as one can see from lemmas 1 and 2, are similar to the case with the non-binding fraud incentive constraint (6). For example, as the lender's incentive to acquire information about the dividend state increases, the borrower attempts to deter information acquisition by giving informational rent first (IIS-5), and then cuts the quantity of collateral trees instead of providing informational rent (IIS-6) to deter information acquisition. In the following, we analyze the interest rate and haircuts because they can respond to a change of some parameter values differently due to the over-collateralization.

As explained above, IIS-4 and IIS-6 loan contracts do not contain informational rent. Thus, the interest rate is the same as $\frac{[\alpha+\eta(1-\alpha)(1-\sigma)]k-\alpha\overline{y}}{(1-\alpha)[1-\eta(1-\sigma)]k}$ for IIS-4 and IIS-6 types, and it increases with η and decreases with α and σ because of their effects on the opportunistic default, similar to the IIS-1 contract. However, in these cases, the fraud cost k affects the interest rate due to the binding fraud incentive constraint (6). More precisely, an increase in the fraud cost k raises the repayment p through the binding fraud incentive constraint (6). This, in turn, increases the loan size q by relaxing the lender's participation constraint (4) or the no-information acquisition constraint (5). Then, as one can see from (4) and (5), an increase in k raises p more than q, so the interest rate of IIS-4 (IIS-6) increases with k. One interesting result is that if Lucas trees are a safe asset, i.e., $\sigma = 1$ so $\overline{y} = y$, the interest rate is $\frac{\alpha(k-\overline{y})}{(1-\alpha)k} < 0$ given $k < \overline{y}$. Then, by continuity, if $\sigma \approx 1$, then the interest rate of IIS-4 and IIS-6 debt contracts will be negative, which sheds light on some evidence on the negative interest rate of a repo contract in which safe assets are traded.

The interest rate of IIS-5 is slightly more complicated as $\frac{[\alpha+\eta(1-\alpha)](k-\alpha\overline{y})(1-\sigma)a-(1-\alpha)[1-\eta(1-\sigma)]\gamma}{(1-\alpha)\{(1-\eta)(k-\alpha\overline{y})(1-\sigma)a+[1-\eta(1-\sigma)]\gamma\}}$ because of the informational rent. The effects of parameters such as η, γ, and σ on the interest rate are the same as those for IIS-2 by similar reasoning. The interest rate also increases with the fraud cost k because of its effect on the value of repayment p and the loan size q as explained above. However, the effects of α on the interest rate are unclear, in contrast to the IIS-2 type. As we explained above, when the no-information acquisition constraint binds, an increase in α has two opposing effects on the interest rate. First, it lowers the possibility of opportunistic default, which pushes down the interest rate. Second, it tightens the binding no-information acquisition constraint, which pushes up the interest rate. In the IIS-2 case, the second effect dominates the first one. In the IIS-5 type, there is a third effect: An increase in α lowers the repurchase price, p, through the binding fraud incentive constraint (6), which works as a force of lowering the interest rate. Combined together, the effects of changing α on the interest rate are ambiguous.

We now analyze the haircuts of IIS loan contracts. First, when the fraud incentive constraint (6) does not bind due to a sufficiently high fraud cost k as IIS-1 to IIS-3 types,

$p = \overline{y}a'$. Therefore, the interest rate on loans $r = \frac{p-q}{q}$ and the haircut $\theta = \frac{\overline{y}a'-q}{\overline{y}a'}$ are basically the same indicators, although the denominators of each variable are different. However, when the fraud incentive constraint (6) binds, the secured loan is over-collateralized, i.e., $p < \overline{y}a'$. Thus, it seems worthwhile to spend a little time on the haircut in this case. From lemma 2, we obtain $\theta = \frac{\overline{y}-k}{\overline{y}}$ for IIS-4 and IIS-6 types, so it only depends on the fraud cost k. The haircut of the IIS-5 type is given as $\theta = 1 - \frac{(1-\eta)(k-\alpha\overline{y})(1-\sigma)a+[1-\eta(1-\sigma)]\gamma}{(1-\sigma)[1-\eta(1-\sigma)]\overline{y}a}$, and it increases with η and α and decreases with γ and k. Note that when loans are over-collateralized, the haircut and interest rate of secured loan contracts can respond differently to shocks on some parameters. For example, the interest rate of the IIS-4 (IIS-6) type changes in response to changes in parameters such as η, α, and σ, but the haircut does not change. In the IIS-5 type case, the haircut increases with α while the effects of α on the interest rate are uncertain. In particular, as the fraud cost k increases, the interest rates rise, but the haircuts fall when loans are over-collateralized (IIS-4 to IIS-6).

3.1.2 Information sensitive loan contracts (IS)

If the information acquisition cost γ is too low, such that the lender has a high incentive to produce information, then it could be too costly for the borrower to deter information acquisition with IIS contracts. Instead, an offer that induces the lender to acquire information about the dividend state may be a better option. Notice that the borrower can always decide not to trade with the lender, which gives the payoff of $V = \overline{y}a$ in expectation. Thus, if the information acquisition cost, γ, is not low enough, then the information sensitive (IS) contract is not profitable, or it may not even be feasible. Here, we focus on IS loan contracts that give a higher payoff to the borrower than the no trade option and impose the necessary conditions for those types of contracts to exist.

Under IS loan contracts, the lender trades with the borrower only if the dividend state is good, so the borrower cannot default opportunistically. Thus, the borrower's expected payment to the lender is $\sigma[(1-\alpha)p + \alpha y a']$. Similar to IIS loan contracts, this expected payment should not be higher than the fraud cost ka' to prevent the borrower from engaging in fraud, which generates the fraud incentive constraint for an IS loan contract. Then, the borrower's maximized value under an IS loan contract, V_{IS}, is obtained by solving the following maximization problem:

$$V_{IS} = \underset{q,p,a'}{Max} \{\sigma[mq - (1-\alpha)p - \alpha y a'] + \overline{y}a\} \tag{10}$$

subject to

$$-\sigma q + (1-\alpha)\sigma p + \alpha \overline{y} a' - \gamma \geq 0 \tag{11}$$
$$(1-\sigma)q - (1-\eta)(1-\alpha)(1-\sigma)p - \gamma \geq 0 \tag{12}$$
$$ka' - (1-\alpha)\sigma p - \alpha \overline{y} a' \geq 0 \tag{13}$$
$$ya' - p \geq 0 \tag{14}$$
$$a - a' \geq 0 \tag{15}$$
$$q, p, a' \geq 0. \tag{16}$$

The objective function (10) is the sum of the borrower's expected surplus from trading and the expected value of the tree holdings. The inequality (11) is the lender's non-negative profit constraint with information acquisition. (12) is the information acquisition constraint that induces the lender to acquire information about the dividend state of the trees, and (13) is the fraud incentive constraint that states that the benefit from fraud should not be higher than the cost of producing fraudulent trees. The inequality (14) is the incentive constraint for the borrower to make repayments on the loan, and (15) and (16) are the feasibility constraints. Notice, from (14), that the borrower has an incentive to repay the loan instead of abandoning the collateral trees as long as $p \leq ya'$ because the lender trades with the borrower only if the dividend state is good.

As explained above, when the lender acquires information about the dividend state, the borrower must compensate the lender for the information acquisition cost to make the lender accept the offer. Therefore, for IS loan contracts to exist, the information acquisition cost, γ, must be sufficiently low. In other words, there is no $\{q, p, a'\}$ that satisfies (11) - (16) if γ is sufficiently high. For example, if $\gamma > [\alpha + (1-\alpha)\eta](1-\sigma)\overline{y}a$, then (11), (12), (14), and (15) cannot be satisfied at the same time. Moreover, V_{IS} must be higher than $\overline{y}a$ because no trade is always a feasible option. Given these arguments, the next lemma describes information sensitive (IS) secured loan contracts.

Lemma 4 *Define the cutoff level of* γ *as* $\gamma^*_{IS} \equiv [\eta k + \alpha(1-\eta)\overline{y}](1-\sigma)a$. *Then, an IS secured loan contract has one of the following forms:*

1) *(IS-1) If* $\overline{y} \leq k$ *and* $\gamma \leq Min\left\{[\alpha + (1-\alpha)\eta](1-\sigma)\overline{y}a, \frac{(m-1)\overline{y}a}{m}\right\}$, *then* $q = ya - \frac{\gamma}{\sigma}$, $p = ya$, $a' = a$, *and* $V_{IS} = m(\overline{y}a - \gamma)$.
2) *(IS-2) If* $\alpha\overline{y} \leq k < \overline{y}$ *and* $\gamma \leq Min\left\{\gamma^*_{IS}, \frac{(m-1)ka}{m}\right\}$, *then* $q = \frac{ka-\gamma}{\sigma}$, $p = \frac{(k-\alpha\overline{y})a}{(1-\alpha)\sigma}$, $a' = a$, *and* $V_{IS} = (m-1)ka - m\gamma + \overline{y}a$.
3) *Otherwise, IS secured loan contracts are not feasible or are worse than no trade, i.e.,* $a' = 0$.

Similar to IIS loan contracts, if the fraud cost k is lower than $\alpha\overline{y}$, then IS loan contracts are not feasible, and when k is in a moderate range, as $\alpha\overline{y} \leq k < \overline{y}$ (IS-2 type), the IS loan contract is over-collateralized. The difference from IIS loan contracts is that for IS loan contracts to be feasible or profitable, the information acquisition cost γ should be sufficiently low, as explained above. More precisely, if $\gamma > [\eta Min\{k, \overline{y}\} + \alpha(1-\eta)\overline{y}](1-\sigma)a$, then IS loan contracts are not feasible because such contracts cannot satisfy the lender's participation constraint with costly information production by the lender (see the proof of Lemma 4). On the other hand, if $\gamma > \frac{Min\{k,\overline{y}\}(m-1)a}{m}$, then IS loan contracts are worse than no trade even though they may be feasible.

Given the information acquisition, trading occurs only if the dividend state is good, and hence, the lender's acceptance of the offer reveals this information. Thus, the expected value of the collateral is ya', and the ratio of over-collateralization and the haircut are defined as $\frac{ya'-p}{ya'}$ and $\frac{ya'-q}{ya'}$, respectively. First, the IS-1 loan contract is not over-collateralized, and the over-collateralization ratio of the IS-2 loan contract is given as $\frac{\overline{y}-k}{(1-\alpha)\overline{y}}$, which increases with α and decreases with k based on similar reasoning as IIS loan contracts. Second, the haircut is $\frac{\gamma}{\overline{y}a}$ for IS-1 and $\frac{\overline{y}a-ka+\gamma}{\overline{y}a}$ for IS-2. Thus, the haircut increases with the information acquisition cost γ for both cases but decreases with the fraud cost k for the IS-2 loan contract. As one can see, the over-collateralization ratio and haircuts do not depend on η because opportunistic default is not possible.

Next, under IS loan contracts, there is no reason for the borrower to provide informational rent to deter information acquisition, and the expected surplus of the lender is zero. The interest rate on the loan, $\frac{\gamma}{\overline{y}a-\gamma}$ for IS-1 and $\frac{(1-\alpha)\gamma-\alpha(\overline{y}-k)a}{(1-\alpha)(ka-\gamma)}$ for IS-2, respectively, represents compensation for the cost of information acquisition. Thus, the interest rate increases with the information acquisition cost γ, in contract to IIS loan contracts. Note that the interest rate on the IS-2 contract can be negative if $\gamma \approx 0$. Because trading occurs only if the dividend state is good under IS loan contracts, Lucas trees can be interpreted as a safe asset if $\gamma \approx 0$. In this sense, the possibility of a negative interest rate on the IS-2 loan contract is consistent with the result that the interest rate on IIS loan contracts can be negative when the loans are over-collateralized and the collateral assets are sufficiently safe. Furthermore, similar to IIS loan contracts, the correlation between the interest rate and haircuts on the IS-1 loan contract is positive, but the interest rate and haircut change in opposite directions for the IS-2 loan contract when k changes.

Finally, the borrower's maximized value V_{IS} does not depend on the counter-party risk α and η under IS loan contracts. The intuition is as follows. First, because the borrower cannot default opportunistically under an IS loan contract, the probability η that the borrower receives information about the dividend state at the beginning of $t = 1$ does not matter for the terms of the contract and the maximized value V_{IS}. Second, under the IS-1 contract, the probability α that the borrower does not receive endowments does not affect the terms of the contract because trading occurs when the dividend state is good and $p = ya'$. However, the terms of the IS-2 contract with the binding fraud incentive constraint (13) depend on α because of the effects α on the fraud incentive. More precisely, whenever the loan is over-collateralized, i.e., $p < ya'$, it is costlier for the borrower to cede collateral trees a' than to make repayment p. Therefore, as α increases, the borrower's expected repayment to the lender, $\sigma\left[(1-\alpha)p + \alpha y a'\right]$, increases, thus raising the borrower's incentive to produce fraudulent trees. To satisfy the binding fraud incentive constraint (13), the repayment p falls. Therefore, an increase of α has two opposing effects on the borrower's expected payment: It increases the probability of losing the ownership of collateral trees, but it lowers the size of the repayment. These two effects are exactly cancelled out, so the initial loan size q and the borrower's maximized value V_{IS} do not change with respect to a change in α.

3.1.3 Induce information acquisition or not?

At $t = 0$, the borrower will compare the maximized values V_{IIS} and V_{IS} given in equations (3) and (10), respectively, and choose the one that gives the higher surplus. Thus, the borrower's maximized value with secured loans is given as $V = Max\{V_{IIS}, V_{IS}\}$. If $V_{IIS} \geq V_{IS}$, then an IIS loan contract dominates an IS loan contract, and vice versa. The type of optimal secured loan contract that the borrower would choose among secured loan contracts depends on the information acquisition cost γ, the fraud cost k, and the exogenous default probability α, and is described in the following proposition.

Proposition 1 *The type of optimal secured loan contract is as follows:*

1) Suppose $\overline{y} \leq k$.

1-a) If $\alpha \leq \alpha^*$, there is $\widehat{\gamma}_1 < \gamma^*_{IIS}$, such that the optimal secured loan contract is i) IS-1 for all $\gamma \in (0, \widehat{\gamma}_1)$, ii) IIS-2 for all $\gamma \in [\widehat{\gamma}_1, \gamma^*_{IIS})$, and iii) IIS-1 for all $\gamma \geq \gamma^*_{IIS}$.

1-b) If $\alpha > \alpha^*$, there is $\widetilde{\gamma}_1 < \gamma^*_{IIS}$, such that the optimal secured loan contract is i) IS-1 for all $\gamma \in (0, \widetilde{\gamma}_1)$, ii) IIS-3 for all $\gamma \in [\widetilde{\gamma}_1, \gamma^*_{IIS})$, and iii) IIS-1 for all $\gamma \geq \gamma^*_{IIS}$.

2) Suppose $[1 - \eta(1 - \alpha)(1 - \sigma)]\overline{y} \leq k < \overline{y}$.

2-a) If $\alpha \leq \alpha^*$, there is $\widehat{\gamma}_2 < \gamma^*_{IIS}$, such that the optimal secured loan contract is i) IS-2 for all $\gamma \in (0, Min\{\widehat{\gamma}_2, \gamma^{**}_{IS}\})$, ii) IIS-2 for all $\gamma \in [Min\{\widehat{\gamma}_2, \gamma^{**}_{IS}\}, \gamma^*_{IIS})$, and iii) IIS-1 for all $\gamma \geq \gamma^*_{IIS}$.

2-b) If $\alpha > \alpha^*$, then there is $\widetilde{\gamma}_2 < \gamma^*_{IIS}$, such that the optimal secured loan contract is i) IS-2 for all $\gamma \in (0, Min\{\widetilde{\gamma}_2, \gamma^{**}_{IS}\})$, ii) IIS-3 for all $\gamma \in [Min\{\widetilde{\gamma}_2, \gamma^{**}_{IS}\}, \gamma^*_{IIS})$, and iii) IIS-1 for all $\gamma \geq \gamma^*_{IIS}$.

3) Suppose $\alpha\overline{y} \leq k < [1 - \eta(1 - \alpha)(1 - \sigma)]\overline{y}$.

3-a) If $\alpha \leq \alpha^{**}$, there is $\widehat{\gamma}_3 < \gamma^{**}_{IIS}$, such that the optimal secured loan contract is i) IS-2 for all $\gamma \in (0, \widehat{\gamma}_3)$, ii) IIS-5 for all $\gamma \in [\widehat{\gamma}_3, \gamma^{**}_{IIS})$, and iii) IIS-4 for all $\gamma \geq \gamma^{**}_{IIS}$.

3-b) If $\alpha > \alpha^{**}$, there is $\widetilde{\gamma}_3 < \gamma^{**}_{IIS}$, such that the optimal secured loan contract is i) IS-2 for all $\gamma \in (0, \widetilde{\gamma}_3)$, ii) IIS-2 for all $\gamma \in [\widetilde{\gamma}_3, \gamma^{**}_{IIS})$, and iii) IIS-4 for all $\gamma \geq \gamma^{**}_{IIS}$

4) If $k < \alpha\overline{y}$, then a secured loan contract is not feasible.

Although the specific type of optimal secured loan contract depends on the fraud cost k in proposition 1, it is more likely that IIS loan contracts dominate IS loan contracts as γ increases. The intuition is in line with our earlier observations. As one can see from lemmas 1 - 4, V_{IIS} weakly increases with the information acquisition cost, γ, while V_{IS} weakly decreases with γ. This is because an increase in γ relaxes the no-information acquisition constraint

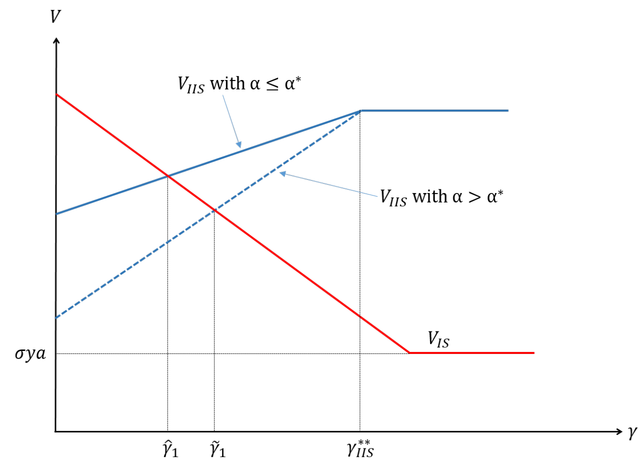

Figure 2: V_{IIS} and V_{IS} with respect to γ when $\overline{y} \leq k$

(5) for IIS loan contracts, while it means a higher information acquisition cost for IS loan contracts. Thus, the borrower makes an offer that induces the lender to acquire information only if the information acquisition cost γ is sufficiently low. This is illustrated in Figure 2, which describes V_{IIS} and V_{IS} with respect to γ when $\overline{y} \leq k$, and the maximized value of the borrower under secured loan contracts, V, is given by the upper line of both graphs. In the following, we analyze some properties of secured loan contracts that were not examined in the previous subsection.

First, as k decreases, the fraud incentive starts to matter for IS loan contracts first as one can see from proposition 1. This is because under IIS loan contracts, the borrower can default opportunistically, which lowers the expected repayment on the loan. This, in turn, implies less benefit from producing fraudulent trees. Note that if $\eta = 0$, such that the borrower cannot default opportunistically, then the second case in proposition 1 with $[1 - \eta(1 - \alpha)(1 - \sigma)]\overline{y} \leq k < \overline{y}$ disappears, and the fraud incentive constraint binds for both IIS loan contracts and IS loan contracts when $k < \overline{y}$.

Second, consider two limiting cases: 1) $\gamma \to \infty$ and 2) $\gamma \to 0$. When $[1 - \eta(1 - \alpha)(1 - \sigma)]\overline{y} \leq k$, the fraud incentive constraint of IIS loan contracts does not bind. In that case, as $\gamma \to \infty$, the IIS-1 loan type is optimal among secured loan contracts and $V = [m - (m-1)(1-\alpha)(1-\sigma)\eta]\,\overline{y}a$. On the other hand, $\lim_{\gamma \to 0} V = m\overline{y}a$ under the IS-1 loan contract if $\overline{y} \leq k$, or $\lim_{\gamma \to 0} V = (m-1)ka + \overline{y}a$ under the IS-2 loan contract if $k < \overline{y}$. In either case, the borrower obtains a higher surplus when $\gamma \to 0$ than when $\gamma \to \infty$. The reason is as follows. Under the IIS-1 loan contract, the borrower can default opportunistically if he/she receives a private signal about the dividend state. Thus, the borrower has to compensate the lender for taking this risk. When the lender acquires information, there is no asymmetric information. Thus, the borrower cannot default in an opportunistic way, and the borrower does not need to pay for the opportunistic default risk. Note that if $\eta = 0$, such that the borrower does not receive a private signal, then $\lim_{\gamma \to \infty} V = \lim_{\gamma \to 0} V$. Next, when $k < [1 - \eta(1-\alpha)(1-\sigma)]\overline{y}$, the fraud incentive constraints of IIS and IS contracts bind. In this case, V_{IIS} under the IIS-4 type does not depend on η because the loan size q and the expected payment $(1-\alpha)[1-(1-\sigma)\eta]p + \alpha\overline{y}a'$ are pinned down as ka through the binding fraud incentive constraint (6), and $\lim_{\gamma \to \infty} V = \lim_{\gamma \to 0} V$, even though $\eta > 0$.

Third, if the fraud cost k is sufficiently high, such as $k \geq \overline{y}$, the fraud incentive constraints - equation (6) for IIS loan contracts and (13) for IS loan contracts - do not bind. In that case, over-collateralization does not exist, as one can see from lemmas 1 and 4, and the interest rate and haircut are technically the same indicator. Therefore, the default risk and the lender's information acquisition incentive cannot explain the existence of over-collateralization in secured loan contracts in the model. Instead, interest rates and haircuts reflect the default risk and the information acquisition incentive.

On a related point, it seems worthwhile to discuss the recent work on repo contracts in the context of information acquisition presented by Dang, Gorton, and Holmström (2012), who derived over-collateralization in a similar economic environment. However, they assumed that the interest rate should be zero, and this assumption drives a positive haircut in secured loan contracts. Given that the repo rate is zero, haircuts are the same as the over-collateralization ratio, which is defined as the percentage difference between the collateral value and the value of repayment on the loan in this paper. More precisely, a lender in Dang, Gorton, and Holmström (2012) may have to sell collateral assets at a discounted price to a third party because the third party can learn the exact value of the collateral assets at some cost similar to the information acquisition technology in our model. Clearly, when a borrower and a lender enter into a secured loan contract, the borrower must compensate the lender in some shape or form for the possibility that the lender may have to sell the collateral asset at a discounted price. However, Dang, Gorton, and Holmström (2012) did not allow a positive interest rate in their model, so the compensation was embodied in a secured loan contract in the form of over-collateralization, which is the haircut in their model. On the other hand, we extend the model to allow the lender can provide interest on a secured loan and show that over-collateralization occurs not because of information acquisition incentive but because of the threat of fraudulent practices in financial markets.

Deriving over-collateralization endogenously is important to better understand repo markets. In particular, Baklanova et al. (2017) document a negative correlation between haircuts and interest rates when dealers borrow cash using U.S. treasuries as collateral. However, if there is no over-collateralization or over-collateralization is introduced into the model exogenously, haircuts and interest rates always move in the same direction in response to any source of risk based on the definitions of both variables. By endogenizing over-collateralization using a moral hazard problem, our model provides the theoretical explanation about the possibility of a negative correlation between haircuts and interest rates when loans are over-collateralized. In particular, for instance, the interest rate increases and the haircut falls as the fraud cost k increases when loans are over-collateralized. Additionally, under the IIS-5 loan contract, the interest rate falls while the haircut rises with respect to the exogenous default probability α under appropriate parameter values.[7] At the same time, our model permits the positive correlation between interest rates and haircuts in response to changes in various parameters, thus supporting the empirical findings of earlier works such as Benmelech, Garmaise, and Moskowitz (2005).

Finally, if $k < \alpha\overline{y}$, secured loan contracts are not feasible because of the threat of fraud. Furthermore, trees cannot be traded in a direct asset sale because an asset sale is a special case of a secured loan contract where $\alpha = 1$. Thus, trees are illiquid when the fraud incentive problem is severe similar to Li, Rocheteau, and Weill (2012), although a secured loan contract and an asset sale were treated equivalently in their model. The difference between Li, Rocheteau and Weill (2012) and our model is that when the fraud incentive problem is severe, the entire trees are not traded in our model, while only a fraction of an illiquid asset is not traded in Li, Rocheteau and Weill (2012).

[7] An increase in α raises the haircut of the IIS-5 loan contract, but its effects on the interest rate are uncertain. However, the interest rate of the IIS-5 loan contract falls as α increases if 1) the trees are sufficiently safe, as $\sigma > \frac{(1-\eta)(k-\alpha\overline{y})^2 a-(\overline{y}-k)(1-\eta)\gamma}{(\overline{y}-k)\eta\gamma+(1-\eta)(k-\alpha\overline{y})^2 a}$, or 2) if the default risk is sufficiently low, i.e., $\eta \approx \alpha \approx 0$ and $k < \frac{\left(\sqrt{9-4\sigma}-1\right)\sigma}{2}$.

3.2 Secured loan vs. Asset sale

Thus far, we have focused on secured loan contracts. However, as argued previously, the borrower can potentially sell trees to the lender on the spot in period 0. This asset sale can be interpreted as a special case of a secured loan contract with $\alpha = 1$ because, in that case, the borrower can never be able to repay the loan, so the lender always seizes the collateral trees. Thus, the borrower's maximized value with an asset sale, which is denoted as $V_{\alpha=1}$, can be obtained by plugging $\alpha = 1$ into proposition 1. Because trees are illiquid when $k < \alpha\overline{y}$, we focus on the cases where $k \geq \alpha\overline{y}$.

As one can see from lemmas 2 and 4, the necessary conditions to have binding fraud incentive constraints are $\alpha\overline{y} \leq k < [1 - \eta(1 - \alpha)(1 - \sigma)]\overline{y}$ for IIS-type loan contracts and $\alpha\overline{y} \leq k < \overline{y}$ for IS-type loan contracts. If $\alpha = 1$, neither of these two conditions can be satisfied. Therefore, if the optimal secured loan contract is over-collateralized, then an asset sale is not a feasible option for tree trading, so a secured loan is the optimal contract. More precisely, by over-collateralizing secured loan contracts, which is not possible under a direct sale of trees, the borrower can give the lender a signal about the authenticity of the collateral trees, thus circumventing the fraud incentive problem. From a technical point of view, this result implies that it suffices to look at the first case of proposition 1, where $\overline{y} \leq k$ and hence the fraud incentive does not exist, when comparing secured loan contracts with tree sales because in the other cases, tree sales are not feasible. The next lemma shows a property of the IS-1 loan contract that is a general property of information sensitive (IS) loan contracts, which provides a useful intermediate step.

Lemma 5 *Suppose $k \geq \overline{y}$ and $\gamma < Min\{\widehat{\gamma}_1, \widetilde{\gamma}_1\}$ given $\alpha = \alpha_0$. Then, IS-1 is the optimal secured loan contract for all $\alpha \geq \alpha_0$, and the borrower is indifferent between secured loan contracts and a direct sale of trees.*

In the proof of Lemma 5, we show that $\widehat{\gamma}_1 \leq \widetilde{\gamma}_1$ if and only if $\alpha \leq \alpha^*$. Therefore, when $k \geq \overline{y}$ given $\alpha = \alpha_0$, the IS-1 type is the optimal secured loan contract for all $\gamma < Min\{\widehat{\gamma}_1, \widetilde{\gamma}_1\}$. Then, Lemma 5 states that the IS-1 loan contract is also optimal for all $\alpha \geq \alpha_0$, which implies that a direct sale of trees is also information sensitive because a tree sale equals a secured loan contract with $\alpha = 1$. Furthermore, under the IS-1 loan contract, the borrower's maximized value does not depend on α (see lemma 4). Therefore, whenever the IS-1 loan contract is optimal among secured loan contracts, the borrower is indifferent between a secured loan contract and a tree sale. In the following, we focus on the case in which the borrower strictly prefers one type to the other.

First, suppose that $\eta = 0$, so the borrower cannot default on the loan in an opportunistic way in period $t = 1$. Then, under the IIS-1 loan contract in which the no-information acquisition constraint does not bind, the borrower's maximized value, V_{IIS}, does not depend on α, similar to the case with the information sensitive (IS) loan contracts. On the other hand, V_{IIS} strictly decreases with α under IIS loan contracts with the binding no-information acquisition constraint. This means that $V = Max\{V_{IIS}, V_{IS}\}$ is weakly decreasing in α, so secured loan contracts are either better than or equivalent to tree sales. Thus, secured loan contracts are always optimal contracts, similar to previous studies such as Gottardi, Maurin, and Monnet (2015), and Parlatore (2017). The intuitive explanation for this finding is as follows. Under a secured loan contract, the borrower takes the collateral trees if he/she makes a repayment on the loan at $t = 1$. Hence, the lender has less incentive to acquire costly information about the dividend state because the dividend state matters only if the lender seizes the collateral trees. Therefore, the borrower can relax the no-information acquisition constraint by offering a secured loan contract rather than a direct sale of trees.

On the other hand, if the borrower receives private information about the dividend state with a positive probability $\eta > 0$, the borrower can default on the loan in a profitable way in period $t = 1$ even though he/she receives the endowment. The borrower must compensate the lender for taking the risk of opportunistic default on the loan contract. Because the borrower can default opportunistically only if he/she receives the endowment e, an increase in the probability α that the borrower will not receive the endowment lowers the possibility that the borrower will take advantage of private information under an IIS loan contract.

Therefore, the borrower's maximized value, V, under the IIS-1 loan contract, in which the no-information acquisition constraint (5) does not bind, strictly increases with α. However, when the no-information acquisition constraint binds, an increase in α tightens the binding no-information acquisition constraint, because as α increases, it is more likely that the lender will end up owning the collateral trees. This effect of increasing α dominates the first effect on the opportunistic default possibility, as explained in the previous section. Thus, an increase in α reduces V_{IIS} under IIS loan contracts with the binding no-information acquisition constraint.

To obtain more intuition, note from the first case with $\overline{y} \leq k$ in proposition 1, that if

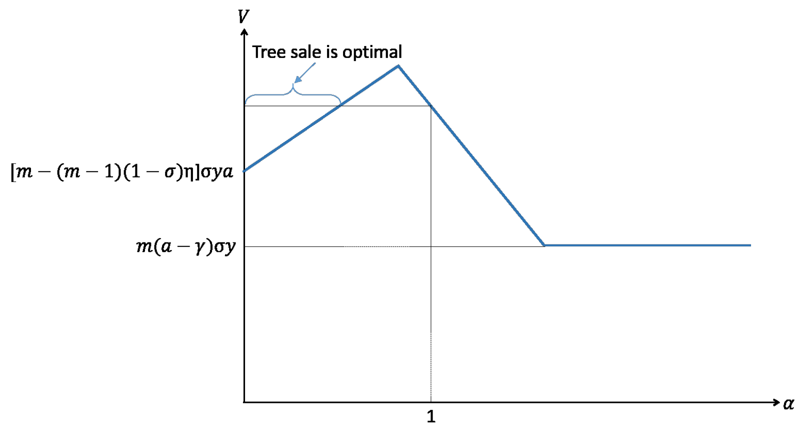

Figure 3: V with respect to α when $\sigma y \leq k$

$\gamma \geq \gamma^*_{IIS}$ at $\alpha = \alpha_0$, then the IIS-1 type is the optimal secured loan contract for all $\alpha \leq \alpha_0$ because $\gamma^*_{IIS} \equiv [\alpha + (1-\alpha)\eta\sigma](1-\sigma)\overline{y}a$, defined in lemma 1, increases with α. Now suppose that there exists $\alpha' \in (0,1)$ such that when $\alpha = \alpha'$, the IIS-1 contract is the optimal secured loan contract and the borrower's maximized value V equals the maximized value with tree sales $V_{\alpha=1}$. Then, a direct sale of trees is optimal for all $\alpha < \alpha_0$ because V_{IIS} under the IIS-1 contract increases with α, as illustrated in Figure 3. Otherwise, secured loan contracts are optimal or the borrower is indifferent between both types of trading arrangements if the secured loan contract is information sensitive. The above analysis leads to the next proposition.

Proposition 2 *1. Suppose $\alpha\overline{y} \leq k < \overline{y}$. Then an asset sale is not feasible, and, hence, secured loan contracts are optimal.*

2. Suppose $\overline{y} \leq k$. 2-a) If $\gamma < Min\{\widehat{\gamma}_1, \widetilde{\gamma}_1\}$, then secured loan contracts and asset sales are information sensitive and the borrower is indifferent between them. 2-b) If $Max\left\{\frac{(m-1)(1-\sigma)\overline{y}a}{m-1+m(1-\sigma)}, [1-\eta(1-\sigma)](1-\sigma)\overline{y}a\right\} \leq \gamma$, then for all $\alpha \in \left[0, \frac{\gamma-[1-\eta(1-\sigma)](1-\sigma)\overline{y}a}{\eta(1-\sigma)^2\overline{y}a}\right)$, tree sales are optimal. 2-c) Otherwise, the borrower prefers secured loan contracts to tree sales because secured loan contracts reduce the lender's information acquisition incentive.

The above proposition shows that secured loan contracts can be the optimal contract for asset trading for two reasons. First, when the fraud incentive problem of misrepresenting the quality of trees is severe, a secured loan contract is optimal because over-collateralization in a secured loan contract mitigates the fraud incentive problem, allowing trees to be tradeable as a medium of exchange. Thus, whenever a secured loan contract is over-collateralized, it must be the optimal contract. Second, even when the fraud incentive problem does not exist, a secured loan contract can still be optimal because it reduces the lender's incentive to acquire costly information about the dividend state, as long as the information acquisition cost γ is neither too high nor sufficiently low.

However, when the lender does not have any incentive to acquire the information because of a high acquisition cost, a secured loan contract only allows the borrower to default in a profitable way whenever possible. Thus, the lender faces the risk of opportunistic default by the borrower. Because the borrower must compensate the lender for taking this risk to make him accept the offer, a secured loan contract can be suboptimal, and it is better for the borrower to sell trees to purchase the lender's goods in period $t = 0$.

Tomura (2016) extends his model to show that an asset sale without a repurchase agreement can be better than a repo if the repo is not exempt from the automatic stay. More precisely, if a borrower defaults on the loan, then the lender cannot dispose of the collateral assets and must hold them until maturity. The optimality of an asset sale in his model, however, is a result of the exogenous assumption that the intrinsic value of collateral assets is lower to the lender than to any other agents. In our model, on the other hand, we show that an asset sale can be optimal even though all agents value the collateral trees symmetrically because of the possibility of opportunistic default that comes from informational frictions, complementing previous studies.

4 Conclusion

When are assets used as collateral or sold directly to raise funds for liquidity needs? In this paper, we construct a simple model to study the effects of costly information acquisition and fraudulent practices on the type of optimal contract used in asset trading. In the model, the dividend of an asset follows a stochastic process, but a lender can acquire private information about the future value of the dividends at a cost. A borrower who owns the asset in the first period has an incentive to produce counterfeit assets at a cost. The model is used to examine the conditions under which secured loan contracts and asset sales are inequivalent, so one or the other emerges as the optimal contract for asset trading. Secured loan contracts can be optimal for two reasons. When the borrower's fraud incentive is severe, over-collateralization reduces the incentive to produce fake assets, making the asset tradable, and, hence, a secured loan contract is optimal. A secured loan contract can also be optimal because it reduces the lender's incentive to acquire costly information. However, under a secured loan contract, the borrower may default opportunistically. Thus, if both the fraud incentive and the information acquisition incentive do not exist, an asset sale can be optimal.

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Appendix: Omitted proofs

Proof of Lemmas 1 - 3. Here, we prove lemmas 1 - 3 together by solving the maximization problem of (3). We define the Lagrangian function for the optimal information insensitive repo contract problem (3) as

$$\begin{aligned} L &= mq - (1-\alpha)[1-(1-\sigma)\eta]p - \alpha\overline{y}a' + \overline{y}a + \lambda_1\left[-q + (1-\alpha)[1-(1-\sigma)\eta]p + \alpha\overline{y}a'\right] \\ &+ \lambda_2\left[-(1-\sigma)q + (1-\eta)(1-\alpha)(1-\sigma)p + \gamma\right] + \lambda_3\left[ka' - (1-\alpha)[1-(1-\sigma)\eta]p - \alpha\overline{y}a'\right] \\ &+ \lambda_4\left[\overline{y}a' - p\right] + \lambda_5\left[a - a'\right] + \lambda_6 q + \lambda_7 p + \lambda_8 a' \end{aligned}$$

where λ_i for $i \in \{1, \ldots 8\}$ are the Lagrange multipliers. The first order conditions are

$$\{q\} : m + \lambda_6 = \lambda_1 + \lambda_2(1-\sigma) \tag{17}$$

$$\{p\} : \lambda_4 - \lambda_7 = (1-\alpha)\left\{(\lambda_1 - \lambda_3 - 1)[1-\eta(1-\sigma)] + \lambda_2(1-\eta)(1-\sigma)\right\} \tag{18}$$

$$\{a'\} : \lambda_5 - \lambda_8 = (\lambda_1 - \lambda_3 - 1)\alpha\overline{y} + \lambda_3 k + \lambda_4\overline{y}. \tag{19}$$

Case 1 (IIS-1). $\lambda_2 = \lambda_3 = 0$

From (17) - (19), we obtain $\lambda_1 > 0$, $\lambda_4 - \lambda_7 > 0$, and $\lambda_5 - \lambda_8 > 0$. Thus, $a' = a$, $p = \overline{y}a$, $q = [1-\eta(1-\alpha)(1-\sigma)]\overline{y}a$, and $V_{IIS} = (m-1)[1-\eta(1-\alpha)(1-\sigma)]\overline{y}a + \overline{y}a$. To have $\lambda_2 = \lambda_3 = 0$, it must be $\gamma \geq [\alpha + (1-\alpha)\eta\sigma](1-\sigma)\overline{y}a$ and $k \geq [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$. ■

Case 2 (IIS-2). $\lambda_2 > 0$, $\lambda_1 = \lambda_3 = 0$

Because (5) binds in this case, $q > 0$ and $\lambda_6 = 0$. Suppose $a' = 0$. Then, $p = 0$ by (7), and (4) cannot be satisfied because $q > 0$. Thus, it must be $a' > 0$ and $\lambda_8 = 0$. Next, from (17) and (18), $\lambda_4 - \lambda_7 = (1-\alpha)[m-1-\eta(m-1+\sigma)]$. Suppose $\eta > \frac{m-1}{m-1+\sigma}$. Then, $\lambda_4 = 0$ and $\lambda_7 > 0$, which implies $q = \frac{\gamma}{1-\sigma}$ by (5). However, from (19), we obtain $\lambda_5 - \lambda_8 < 0$, so $a' = 0$, a contradiction. Thus, we assume that $\eta \leq \frac{m-1}{m-1+\sigma}$ from now on. Then, $\lambda_4 - \lambda_7 \geq 0$. Thus, $p = \overline{y}a'$ and, hence, $\lambda_7 = 0$. From (17) - (19), we obtain

$$\lambda_5 = \overline{y}\left\{(m-1)(1-\eta) - \eta\sigma - \alpha\left[(m-1)(1-\eta) - \eta\sigma + 1\right]\right\}. \tag{20}$$

Thus, $\alpha \leq \frac{(m-1)(1-\eta)-\eta\sigma}{(m-1)(1-\eta)-\eta\sigma+1}$ must hold and, hence, $a' = a$ and $p = \overline{y}a$. Then, from the binding (5), $q = (1-\eta)(1-\alpha)\overline{y}a + \frac{\gamma}{1-\sigma}$, and the maximized value is given as $V_{IIS} = m\left[(1-\eta)(1-\alpha)\overline{y}a + \frac{\gamma}{1-\sigma}\right] + (1-\alpha)(1-\sigma)\eta\overline{y}a$. Finally, to have $\lambda_1 = \lambda_3 = 0$, it must be

$\gamma \leq [\alpha + (1-\alpha)\eta\sigma](1-\sigma)\overline{y}a$ and $k \geq [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$. ∎

Case 3 (IIS-3). $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$

Because $\lambda_2 > 0$, it must be $q > 0$ by (5) so $\lambda_6 = 0$. Further, if $a' = 0$, then $p = 0$ by (7), and (4) cannot satisfied. Thus, $a' > 0$ and $\lambda_8 = 0$ must hold. Next, from (17) - (19), we obtain

$$\lambda_4 - \lambda_7 = (1-\alpha)\left[(m-1)(1-\eta) + \eta\sigma(\lambda_1 - 1)\right] \tag{21}$$

$$\lambda_5 = \overline{y}\left\{\begin{array}{c} (m-1)(1-\eta) - \eta\sigma \\ -\alpha[1 + (m-1)(1-\eta) - \eta\sigma] + \lambda_1[\alpha + (1-\alpha)\eta\sigma] \end{array}\right\} + \lambda_7\overline{y} \tag{22}$$

Suppose $\lambda_7 \geq 0$, so $p = 0$ and $\lambda_4 = 0$. Then, it must be $\lambda_1 \leq 1 - \frac{(m-1)(1-\eta)}{\eta\sigma}$ to satisfy (21). Then, $\lambda_5 - \lambda_8 < 0$ by (21) and (22), so $a' = 0$, which is a contradiction. Thus, it must be $\lambda_7 = 0$. Now, suppose $\lambda_4 = 0$, which requires $\lambda_1 = 1 - \frac{(m-1)(1-\eta)}{\eta\sigma}$. Then, we obtain $\lambda_5 < 0$ from (22), a contradiction. Thus, it must be $\lambda_4 > 0$ and $p = \overline{y}a'$. Then, from the binding (4) and (5), we obtain $q = \frac{[1-\eta(1-\alpha)(1-\sigma)]\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)}$ and $a' = \frac{\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)\overline{y}}$. Then, we obtain $p = \frac{\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)}$ and $V_{IIS} = \frac{(m-1)[1-(1-\alpha)(1-\sigma)\eta]\gamma}{[\alpha+(1-\alpha)\eta\sigma](1-\sigma)} + \overline{y}a$. Now suppose $\alpha \leq \frac{(m-1)(1-\eta)-\eta\sigma}{(m-1)(1-\eta)-\eta\sigma+1}$. Then, $\lambda_5 > 0$ by (22), and $a' = a$, which is the knife edge case of case 2. Thus, we focus on the case where $a' < a$ which requires $\alpha > \frac{(m-1)(1-\eta)-\eta\sigma}{(m-1)(1-\eta)-\eta\sigma+1}$. Given this condition, one can always find $\lambda_1 \in (0, m)$ that makes $\lambda_4 \geq 0$ and $\lambda_5 = 0$ in (22). Finally, to have $\lambda_3 = 0$, $k \geq [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$ must hold. ∎

Case 4 (IIS-4). $\lambda_2 = 0$, $\lambda_3 > 0$, $p > 0$

Given $p > 0$, it must be $\lambda_7 = 0$. Then, from (17) - (19), we obtain

$$\lambda_4 = (m + \lambda_6 - \lambda_3 - 1)(1-\alpha)[1-\eta(1-\sigma)] \tag{23}$$

$$\lambda_5 - \lambda_8 = \overline{y}(m + \lambda_6 - \lambda_3 - 1)[1-\eta(1-\alpha)(1-\sigma)] + \lambda_3 k. \tag{24}$$

Suppose $\lambda_4 > 0$, so $p = \overline{y}a'$. Because $m + \lambda_6 - \lambda_3 - 1 > 0$ to have $\lambda_4 > 0$, $\lambda_5 - \lambda_8 > 0$. Thus, $a' = a$ and $q = [1-\eta(1-\alpha)(1-\sigma)]\overline{y}a$ by the binding (4) because $\lambda_1 = m + \lambda_6 > 0$. To have $\lambda_3 > 0$, it must be $k = [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$. This is exactly the knife edge case of case 1. Thus, from now on, we assume that $\lambda_4 = 0$, which implies $p < \overline{y}a'$. From (23) and (24), we obtain $\lambda_5 - \lambda_8 = \lambda_3 k > 0$. Thus, $a' = a$ and $\lambda_8 = 0$. Next, from the binding constraint (4), we obtain

$$q = (1-\alpha)[1-\eta(1-\sigma)]p + \alpha\overline{y}a > 0, \tag{25}$$

so $\lambda_6 = 0$. Given $\lambda_3 > 0$, the constraint (6) must bind, which gives

$$p = \frac{(k-\alpha\overline{y})a}{(1-\alpha)[1-\eta(1-\sigma)]}. \tag{26}$$

Thus, it must be $\alpha\overline{y} < k < [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$ to have $p \in (0, \overline{y}a)$. Substituting (26) into (25), we obtain $q = ka$, and $V_{IIS} = (m-1)ka + \overline{y}a$. Finally, to have $\lambda_2 = 0$, it must be $\frac{[\sigma\eta k+(1-\eta)\alpha\overline{y}](1-\sigma)a}{1-\eta(1-\sigma)} \leq \gamma$. ■

Case 5 (IIS-5). $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$

From the binding (5), we obtain $q = (1-\eta)(1-\alpha)p + \frac{\gamma}{1-\sigma} > 0$, so $\lambda_6 = 0$ and $\lambda_2 = \frac{m}{1-\sigma}$ by (17). Further, to satisfy the constraint (4), $a' > 0$ must hold given $q > 0$, so $\lambda_8 = 0$. Now suppose $\lambda_4 > 0$ so $p = \overline{y}a'$ and $\lambda_7 = 0$. Then, from the binding constraint (6), we obtain

$$k = [1-\eta(1-\alpha)(1-\sigma)]\overline{y}. \tag{27}$$

Then, from (18), (19), and (27), we obtain equation (20). Thus, $\alpha \leq \frac{(m-1)(1-\eta)-\eta\sigma}{(m-1)(1-\eta)-\eta\sigma+1}$ must hold and, hence, $a' = a$ and $p = \overline{y}a$. This is exactly the same as case 2 above.

Thus, we focus on the case where $\lambda_4 = 0$ and, hence, $p < \overline{y}a'$ and $\lambda_3 = \frac{(m-1)(1-\eta)-\eta\sigma}{1-\eta(1-\sigma)}$ by (18). If $p = 0$, then $k = \alpha\overline{y}$ by (6). Then, from (19), $\lambda_5 = -\alpha\overline{y}$, which is a contradiction. Therefore, it must be $p > 0$ and $\lambda_7 = 0$. From (18) and (19), we obtain $\lambda_5 \approx k - \frac{m(1-\eta)\alpha\overline{y}}{(m-1)(1-\eta)-\eta\sigma}$. Therefore, $\alpha \leq \frac{[(m-1)(1-\eta)-\eta\sigma]k}{m(1-\eta)\overline{y}}$ must hold, and, then, $a' = a$. Next, from the binding (5) and (6), we obtain $q = \frac{(1-\eta)(k-\alpha\overline{y})a}{1-\eta(1-\sigma)} + \frac{\gamma}{1-\sigma}$ and $p = \frac{(k-\alpha\overline{y})a}{(1-\alpha)[1-\eta(1-\sigma)]}$, and the maximized value of the objective is given as $V_{IIS} = m\left\{\frac{(1-\eta)(k-\alpha\overline{y})a}{1-\eta(1-\alpha)} + \frac{\gamma}{1-\sigma}\right\} - ka + \overline{y}a$. Because $p \in (0, \overline{y}a)$, $\alpha\overline{y} < k < [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$ must hold. Finally, to satisfy the constraint (4) with $\lambda_1 = 0$, it must be $\gamma \leq \frac{[\sigma\eta k+\alpha(1-\eta)\overline{y}](1-\sigma)a}{1-\eta(1-\sigma)}$. ■

Case 6 (IIS-6). $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$

Since $\lambda_2 > 0$ and the constraint (5) binds $q > 0$ and $\lambda_6 = 0$ must hold. Therefore, it must be $a' > 0$ and $\lambda_8 = 0$ to satisfy (4) and (7). Next, the binding constraint (6) gives $ka' = (1-\alpha)[1-\eta(1-\sigma)]p + \alpha\overline{y}a'$. Now suppose $\lambda_4 > 0$ so $p = \overline{y}a'$. If $a' = a$, then it is the knife edge case of case 1. On the other hand, if $a' < a$, then the result is the same as case 3. Therefore, we focus on the case where $\lambda_4 = 0$ and $p < \overline{y}a'$.

From (4) - (6), we obtain $q = \frac{[1-\eta(1-\sigma)]\gamma k}{[\sigma\eta k+\alpha(1-\eta)\overline{y}](1-\sigma)}$, $p = \frac{(k-\alpha\overline{y})\gamma}{[\sigma\eta k+\alpha(1-\eta)\overline{y}](1-\alpha)(1-\sigma)}$, and $a' = \frac{[1-\eta(1-\sigma)]\gamma}{[\sigma\eta k+\alpha(1-\eta)\overline{y}](1-\sigma)}$. Next, because $p \in [0, \overline{y}a')$, it must be $\alpha\overline{y} \leq k < [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$, and, we can assume $\lambda_7 = 0$ without loss of generality. Then, from (17) - (19), we obtain

$$\lambda_1 = \frac{\lambda_3[1-\eta(1-\sigma)] - (m-1)(1-\eta) + \eta\sigma}{\eta\sigma} \tag{28}$$

$$\lambda_5 = \frac{\lambda_3[(1-\eta)\alpha\overline{y} + \sigma\eta k] - (m-1)(1-\eta)\alpha\overline{y}}{\eta\sigma}. \tag{29}$$

Because $\lambda_1 > 0$, then $\lambda_3 > \frac{(m-1)(1-\eta)-\eta\sigma}{1-\eta(1-\sigma)}$ must hold by (28). Then, substituting $\lambda_3 = \frac{(m-1)(1-\eta)-\eta\sigma}{1-\eta(1-\sigma)}$ into (29), we obtain

$$\lambda_5 > \frac{[(m-1)(1-\eta)-\eta\sigma]k - \alpha m(1-\eta)\sigma\overline{y}}{1-\eta(1-\sigma)}.$$

Therefore, if $\alpha \leq \frac{[(m-1)(1-\eta)-\eta\sigma]k}{m(1-\eta)\overline{y}}$, then $a' = a$, which is a knife edge case of case 6. Thus, we focus on the case where $a' < a$, which requires $\alpha > \frac{[(m-1)(1-\eta)-\eta\sigma]k}{m(1-\eta)\overline{y}}$ and $\gamma < \frac{[\sigma\eta k+\alpha(1-\eta)\overline{y}](1-\sigma)a}{1-\eta(1-\sigma)}$ by the definition of a' in this case. Then, the value of the objective is given as $V_{IIS} = \frac{(m-1)[1-\eta(1-\sigma)]\gamma k}{[\eta k+\alpha(1-\eta)y](1-\sigma)\sigma} + \overline{y}a$. ■

Case 7 (No trade). $\lambda_2 = 0$, $\lambda_3 > 0$, $p = 0$

In this case, we obtain, from (6), $ka' = \alpha\overline{y}a'$. Thus, if $a' > 0$ so $\lambda_8 = 0$, it must be $k = \alpha\overline{y}$, and $\lambda_5 = [(m-1)\alpha + \lambda_4]\overline{y} > 0$. Thus, $a' = a$ and $q = ka$, which is the knife edge case of case 4 above. Assume $a' = 0$, and hence, $q = 0$, which means that trees are not traded in period $t = 0$. Then, (19) becomes $-\lambda_8 = (m + \lambda_6 - 1)\alpha\overline{y} - \lambda_3(\alpha\overline{y} - k) + \lambda_4\overline{y}$. Therefore, the necessary condition for this case to exist is $k < \alpha\overline{y}$. ■

By defining $\gamma^*_{IIS} \equiv [\alpha + (1-\alpha)\eta\sigma](1-\sigma)\overline{y}a$, $\gamma^{**}_{IIS} \equiv \frac{[\sigma\eta k+(1-\eta)\alpha\overline{y}](1-\sigma)a}{1-\eta(1-\sigma)}$, $\alpha^* \equiv \frac{(1-\eta)(m-1)-\eta\sigma}{1+(1-\eta)(m-1)-\eta\sigma}$, and $\alpha^{**} \equiv \frac{k[(m-1)(1-\eta)-\eta\sigma]}{m(1-\eta)\overline{y}}$ and reorganizing cases 1-7 above, we obtain the results of lemmas 1 - 3. ■

Proof of Lemma 4. If $a' = 0$, then $p = 0$ by (14). Then, (11) cannot be satisfied. Further, $q > 0$ must hold by the constraint (12). Therefore, it must be $q > 0$ and $a' > 0$. Then, the first-order conditions are

$$\lambda_1 = m + \frac{(1-\sigma)\lambda_2}{\sigma} \tag{30}$$

$$\lambda_4 - \lambda_6 = (1-\alpha)\left[(m-1)\sigma + \eta(1-\sigma)\lambda_2 - \sigma\lambda_3\right] \tag{31}$$

$$\lambda_5 = (\lambda_1 - \lambda_3 - 1)\alpha\overline{y} + k\lambda_3 + y\lambda_4, \tag{32}$$

where λ_1, λ_2, λ_3, λ_4, and λ_5 are the Lagrange multipliers for (11) - (15), respectively, and λ_6 is the Lagrange multiplier for $p \geq 0$. Note, from (13), that $k \geq \frac{(1-\alpha)\sigma p}{a'} + \alpha\overline{y}$. Substituting this condition and (30) into (32), one can show that $\lambda_5 > 0$, and, hence, $a' = a$. Next, because $\lambda_1 > 0$ by (30), the constraint (11) must bind, so

$$q = (1-\alpha)p + \alpha ya - \frac{\gamma}{\sigma}. \tag{33}$$

Case 1 (IS-1). $\lambda_3 = 0$

In this case, $\lambda_4 > 0$ and $\lambda_6 = 0$ by (31), so $p = ya$, $q = ya - \frac{\gamma}{\sigma}$, and $V_{IS} = m(\overline{y}a - \gamma)$. Then, constraints (12) and (13) require $\gamma \leq [\alpha + (1-\alpha)\eta](1-\sigma)\overline{y}a$ and $k \geq \overline{y}$, respectively. Finally, the borrower can always choose not to trade, which gives the borrower the payoff $\overline{y}a$. Thus, it must be $V_{IS} = m(\overline{y}a - \gamma) \geq \overline{y}a$, which requires $\gamma \leq \frac{(m-1)\overline{y}a}{m}$. Combined together, it must be $\gamma \leq Min\left\{[\alpha + (1-\alpha)\eta](1-\sigma)\overline{y}a, \frac{(m-1)\overline{y}a}{m}\right\}$ for this case to be the optimal IS repo contract. ■

Case 2 (IS-2). $\lambda_3 > 0$

Suppose $\lambda_4 > 0$, and, hence, $\lambda_6 = 0$. Then, $p = ya$ and $k = \sigma y$, which is the knife edge case of the previous case. Thus, we focus on the case where $\lambda_4 = 0$ and $p < ya$. From the binding (13), $p = \frac{(k-\alpha\overline{y})a}{(1-\alpha)\sigma}$. Since $p \in [0, ya)$, $\alpha\overline{y} \leq k < \overline{y}$ must hold. From (33), we obtain $q = \frac{ka-\gamma}{\sigma}$. Then, $V_{IS} = (m-1)ka - m\gamma + \overline{y}a$. Finally, to satisfy the constraint (12) and $V_{IS} \geq \overline{y}a$, it must be $\gamma \leq Min\left\{[\eta k + \alpha(1-\eta)\overline{y}](1-\sigma)a, \frac{(m-1)ka}{m}\right\}$ ■

Note that we impose the condition that an IS secured loan contract gives a higher payoff than no trade, i.e., $V_{IS} \geq \overline{y}a$. Thus, except for the above two cases, an IS secured loan contract is either infeasible or worse than no trade, so the borrower would not offer an IS loan contract to the lender. By defining the cutoff level for γ as $\gamma_{IS}^{**} \equiv [\eta k + \alpha(1-\eta)\overline{y}](1-\sigma)a$ and summarizing cases 1 and 2 above, we obtain lemma 4. ■

Proof of Proposition 1. To save space, let $V_{IIS\text{-}i}$, where $i \in \{1, \dots 6\}$, denote the maximized value of the borrower under the IIS-i type loan contracts. For example, $V_{IIS\text{-}1} = [m - (m-1)(1-\alpha)(1-\sigma)\eta]\,\overline{y}a$. Similarly, let $V_{IS\text{-}1}$ and $V_{IS\text{-}2}$ denote the maximized value of the borrower under IS-1 and IS-2 loan contracts, respectively. Then, the optimal secured loan contract can be obtained by comparing V_{IIS} with V_{IS}. The key is that V_{IIS} is weakly increasing in γ while V_{IS} is decreasing in γ, which implies that if $V_{IIS} = V_{IS}$ at $\gamma = \gamma_0$, then $V_{IIS} \gtreqless V_{IS}$ for all $\gamma \gtreqless \gamma_0$.

From lemmas 1 - 4, we can divide analysis into three groups: 1) $\overline{y} \leq k$, 2) $[1 - \eta(1-\alpha)(1-\sigma)]\overline{y} \leq k < \overline{y}$, and 3) $\alpha\overline{y} \leq k < [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$.

Case 1. $\overline{y} \leq k$.

In this case, IIS-1, IIS-2, IIS-3, and IS-1 are candidates for the optimal repo contracts. First, if $\gamma = \gamma^*_{IIS}$, then $V_{IIS\text{-}1} - V_{IS\text{-}1} \approx m\alpha + (1-\alpha)\eta(m\sigma - m + 1) \geq 0$ given $\sigma \geq \frac{m-1}{m}$. Thus, for all $\gamma \geq \gamma^*_{IIS}$, $V_{IIS\text{-}1} \geq V_{IS\text{-}1}$. Note, from lemmas 1 and 4, that

$$V_{IIS\text{-}2} \geq V_{IS\text{-}1} \text{ iff } \gamma \geq \frac{\{m[\alpha + (1-\alpha)\eta] - (1-\alpha)(1-\sigma)\eta\}(1-\sigma)\overline{y}a}{(2-\sigma)m} \equiv \widehat{\gamma}_1 \tag{34}$$

$$V_{IIS\text{-}3} \geq V_{IS\text{-}1} \text{ iff } \gamma \geq \frac{(m-1)[\alpha + (1-\alpha)\eta\sigma](1-\sigma)\overline{y}a}{(m-1)[1-(1-\alpha)(1-\sigma)\eta] + m(1-\sigma)[\alpha + (1-\alpha)\eta\sigma]} \equiv \widetilde{\gamma}_1. \tag{35}$$

Simple algebra shows that $\frac{(m-1)\overline{y}a}{m} > \widetilde{\gamma}_1$. Next, note, from the definition of α^* in lemma 1, that the necessary condition for IIS-2 to be the optimal IIS loan contract is $\alpha \leq \alpha^*$. Then, using the fact that $\widehat{\gamma}_1$ increases with α, it can be verified that $\frac{(m-1)\overline{y}a}{m} > \widehat{\gamma}_1$ for all $\alpha \leq \alpha^*$. Finally, note that $\gamma^*_{IIS} < [\alpha + (1-\alpha)\eta](1-\sigma)\overline{y}a$ by definition of γ^*_{IIS}, and, hence, $\widehat{\gamma}_1$ and $\widetilde{\gamma}_1$ are lower than the value, $[\alpha + (1-\alpha)\eta](1-\sigma)\overline{y}a$. From the above analysis, we can obtain Figure 2, which proves part 1 of proposition 1.[8] ■

Case 2. $[1-\eta(1-\alpha)(1-\sigma)]\overline{y} \leq k < \overline{y}$

In this case, IIS-1, IIS-2, IIS-3, and IS-2 are candidates for the optimal secured loan contracts. Because $V_{IS\text{-}2}$ increases with k and $V_{IS\text{-}2} = V_{IS\text{-}1}$ when $k = \overline{y}$, it can be verified that $V_{IIS\text{-}1} > V_{IS\text{-}2}$ for all $\gamma \geq \gamma^*_{IIS}$. Thus, IIS-1 is optimal for all $\gamma \geq \gamma^*_{IIS}$. Next, from

[8] In Figure 2, we assume that $\frac{(m-1)\overline{y}a}{m} > \gamma^*_{IIS}$. However, it is possible that $\frac{(m-1)\overline{y}a}{m} < \gamma^*_{IIS}$, but it does not affect the result because for all $\gamma > \widehat{\gamma}_1$ or $\gamma > \widetilde{\gamma}_1$, the IS-1 loan contract is strictly dominated by IIS loan contracts

lemmas 1 and 4,

$$V_{IIS\text{-}2} \geq V_{IS\text{-}2} \text{ iff } \gamma \geq \frac{\{\{1-(1-\alpha)[m(1-\eta)+(1-\sigma)\eta]\}\,\overline{y}a+(m-1)ka\}\,(1-\sigma)}{(2-\sigma)m} \equiv \widehat{\gamma}_2$$

$$V_{IIS\text{-}3} \geq V_{IS\text{-}2} \text{ iff } \gamma \geq \frac{(m-1)[\alpha+(1-\alpha)\eta\sigma](1-\sigma)ka}{(m-1)[1-(1-\alpha)(1-\sigma)\eta]+m(1-\sigma)[\alpha+(1-\alpha)\eta\sigma]} \equiv \widetilde{\gamma}_2.$$

In this case, we need to compare $\frac{(m-1)ka}{m}$, $\widehat{\gamma}_2$, and $\widetilde{\gamma}_2$. First, simple algebra shows $\frac{(m-1)ka}{m} > \widetilde{\gamma}_2$. Next, using the fact that $\widehat{\gamma}_2$ increases with α, one can show that $\frac{(m-1)ka}{m} > \widehat{\gamma}_2$ for all $\alpha \leq \alpha^*$. However, in this case, it is possible that γ_{IS}^{**}, defined in lemma 2, is smaller than $\widehat{\gamma}_2$ and $\widetilde{\gamma}_2$. If $\gamma_{IS}^{**} < \widehat{\gamma}_2$, for example, then for all $\gamma \in [\gamma_{IS}^{**}, \gamma_{IIS}^{*})$, IIS-2 is optimal and IS-2 is optimal for all $\gamma < \gamma_{IS}^{**}$, when $\alpha \leq \alpha^*$. On the other hand, if $\gamma_{IS}^{**} \geq \widehat{\gamma}_2$, then for all $\gamma \in [\widehat{\gamma}_2, \gamma_{IIS}^{*})$, IIS-2 is optimal and IS-2 is optimal for all $\gamma < \widehat{\gamma}_2$, when $\alpha \leq \alpha^*$. A similar argument applies to the case where $\alpha > \alpha^*$. Combined together, we obtain part 2 of proposition 1. ■

Case 3. $\alpha\overline{y} \leq k < [1-\eta(1-\alpha)(1-\sigma)]\overline{y}$

In this case, we have to compare IIS-4, IIS-5, and IIS-6 with IS-2. Note that $V_{IIS\text{-}4} > V_{IS\text{-}2}$ for any $\gamma > 0$. Thus, for all $\gamma \geq \gamma_{IIS}^{**}$, IIS-4 is optimal. Next, by definition of $V_{IIS\text{-}5}$, $V_{IIS\text{-}6}$, and $V_{IS\text{-}2}$, we obtain

$$V_{IIS\text{-}5} \geq V_{IS\text{-}2} \text{ iff } \gamma \geq \frac{[k\eta+\alpha(1-\eta)y](1-\sigma)\sigma a}{(2-\sigma)[1-(1-\sigma)\eta]} \equiv \widehat{\gamma}_3$$

$$V_{IIS\text{-}6} \geq V_{IS\text{-}2} \text{ iff } \gamma \geq \frac{(m-1)[k\eta+\alpha(1-\eta)y]k(1-\sigma)\sigma a}{(m-1)[1-\eta(1-\sigma)]k+m[k\eta+\alpha(1-\eta)y](1-\sigma)\sigma} \equiv \widetilde{\gamma}_3.$$

Observe that $\frac{(m-1)ka}{m} > \widetilde{\gamma}_3$. Next, using the fact that $\widehat{\gamma}_3$ increases with α, it can be verified that $\frac{(m-1)ka}{m} > \widehat{\gamma}_3$ for all $\alpha \leq \alpha^{**}$, where α^{**} is defined in lemma 1, so an IIS loan is the IIS-5 type. Next, simple algebra shows $\gamma_{IS}^{*} > Max\{\widehat{\gamma}_3, \widetilde{\gamma}_3\}$, where $\gamma_{IS}^{*} \equiv [\eta k+\alpha(1-\eta)\overline{y}](1-\sigma)a$ as defined in lemma 4, given $k \geq \alpha\overline{y}$. Thus, if $\alpha \leq \alpha^{**}$, an IS repo contract - either IIS-4 or IIS-5 - is optimal for all $\gamma \geq \widehat{\gamma}_3$, and IS-2 is optimal for all $\gamma < \widehat{\gamma}_3$. A similar argument applies to the case where $\alpha > \alpha^{**}$. Combined together, we obtain part 3 of proposition 1. ■

Case 4. $k < \alpha\overline{y}$

From lemmas 1 and 2, one can see that secured loan contracts - either IIS or IS- are not feasible if $k < \alpha\overline{y}$. ■

Finally, by reorganizing the analysis of the above 4 cases, we obtain proposition 1. ■

Proof of Lemma 5. From (34) and (35), it can be verified that $\widehat{\gamma}_1$ and $\widetilde{\gamma}_1$ increase with α. Thus, if $\gamma \leq Min\{\widehat{\gamma}_1, \widetilde{\gamma}_1\}$ when $\alpha = \alpha_0$, then $\gamma < Min\{\widehat{\gamma}_1, \widetilde{\gamma}_1\}$ for all $\alpha \geq \alpha_0$. The next claim provides an intermediate step.

Claim 1 $\widehat{\gamma}_1 \geq \widetilde{\gamma}_1$ *if and only if* $\alpha \geq \alpha^* \equiv \frac{(m-1)(1-\eta)-\eta\sigma}{(m-1)(1-\eta)-\eta\sigma+1}$.
Proof of Claim 1. *From (34) and (35), one can show that* $\widehat{\gamma}_1 \geq \widetilde{\gamma}_1$ *if and only if*

$$\left\{ \begin{array}{c} m[\alpha + (1-\alpha)\eta] \\ -(1-\alpha)(1-\sigma)\eta \end{array} \right\} \left\{ \begin{array}{c} (m-1)[1-(1-\alpha)(1-\sigma)\eta] \\ +m(1-\sigma)[\alpha + (1-\alpha)\eta\sigma] \end{array} \right\} \geq m(m-1)(2-\sigma)[\alpha + (1-\alpha)\eta\sigma]. \tag{36}$$

Let $\phi \equiv -m(1-\alpha)(1-\eta) + 1 - (1-\alpha)(1-\sigma)\eta$. *Then, (36) becomes*

$$\begin{aligned} &(\phi + m - 1)\left\{-\phi + m(2-\sigma)[\alpha + (1-\alpha)\eta\sigma]\right\} \\ &\geq m(m-1)(2-\sigma)[\alpha + (1-\alpha)\eta\sigma] \\ &\Leftrightarrow \phi(1-\sigma)\left\{m\alpha + \eta(1-\alpha)(m\sigma - m + 1)\right\} \geq 0. \end{aligned}$$

Because $\sigma \geq \frac{m-1}{m}$, $\widehat{\gamma}_1 \geq \widetilde{\gamma}_1$ *if and only if* $\phi \geq 0$, *which requires* $\alpha \geq \alpha^* \equiv \frac{(m-1)(1-\eta)-\eta\sigma}{(m-1)(1-\eta)-\eta\sigma+1}$ *by definition of* ϕ. ■

The above claim implies that if $\gamma < Min\{\widehat{\gamma}_1, \widetilde{\gamma}_1\}$, then the IS-1 type is the best among collateralized debt contracts by the first part of proposition 1 when $k \geq \alpha\overline{y}$. Combined together, the IS-1 is the best among collateralized debts for all $\alpha \geq \alpha_0$. Furthermore, the borrower's maximized value does not depend on α under the IS-1 loan contract (see lemma 4). Therefore, whenever the IS-1 loan contract is optimal among secured loan contracts, the borrower is indifferent between a secured loan contract and a tree sale. ■

Proof of Proposition 2. We already proved the first part in the main body and lemma 3 proves part 2-a). Furthermore, once we prove part 2-b) of proposition 2, then part 2-c) is straightforward. Thus, we focus on the proof of part 2-b) of proposition 2 here. As one can see from proposition 1, the type of loan contract with $\alpha = 1$ can be IIS-1, IIS-3, or IS-1 when $\overline{y} \leq k$. For tree sales to be optimal, the IIS-1 loan contract should be the best among secured loan contracts because if another type of secured loan contract is the best among secured loan contracts, then a loan contract always yields a higher payoff than tree sales (or at least the same payoff as tree sales). Thus, we compare the borrower's maximized value V with the IIS-1 loan contract and the maximized value with tree sales, $V_{\alpha=1}$. For the IIS-1 to be the best loan contract among secured loan contracts, it must be $\gamma \geq \gamma^*_{IIS} \equiv [\alpha + (1-\alpha)\eta\sigma]\,(1-\sigma)\overline{y}a$, which implies $\alpha \leq \frac{\gamma - \eta(1-\sigma)\sigma\overline{y}a}{(1-\eta\sigma)(1-\sigma)\overline{y}a}$. Then, it must be

$$\gamma > \eta(1-\sigma)\sigma\overline{y}a, \tag{37}$$

because $\alpha \geq 0$. Now, we consider three possible cases for the type of loan contract with $\alpha = 1$, which is the same as direct sales of trees.

First, suppose that the type of loan contact with $\alpha = 1$ is the IIS-1. Then, tree sales are optimal because V is increasing in α. When $\alpha = 1$, $\gamma^*_{IIS} = (1-\sigma)\overline{y}a$. Thus, the necessary condition for this case is

$$\gamma \geq (1-\sigma)\overline{y}a. \tag{38}$$

Second, suppose that the type of loan contract with $\alpha = 1$ is the IIS-3, which requires $\gamma \in [\widetilde{\gamma}_1, \gamma^*_{IIS})$ with $\alpha = 1$, and the borrower's maximized value is given by $V_{\alpha=1} = \frac{(m-1)\gamma}{1-\sigma} + \overline{y}a$. By substituting $\alpha = 1$ into the definitions of $\widetilde{\gamma}_1$, whose form is given in equation (35), and γ^*_{IIS}, we obtain the necessary condition for the IIS-3 to be the optimal secured loan contract with $\alpha = 1$ as

$$\frac{(m-1)(1-\sigma)\overline{y}a}{m-1+m(1-\sigma)} \leq \gamma < (1-\sigma)\overline{y}a. \tag{39}$$

Next, the borrower's maximized value, V, under the IIS-1 loan contract is $V_{IIS\text{-}1} = (m-1)[1-\eta(1-\alpha)(1-\sigma)]\overline{y}a + \overline{y}a$, and a tree sale delivers the payoff $V_{\alpha=1} = \frac{(m-1)\gamma}{1-\sigma} + \overline{y}a$ to the borrower. Thus, a tree sale is better than a secured loan contract, so it is optimal only if $\alpha \leq \frac{\gamma - [1-\eta(1-\sigma)](1-\sigma)\overline{y}a}{\eta(1-\sigma)^2\overline{y}a}$. Since $\alpha \geq 0$, the necessary condition is

$$[1-\eta(1-\sigma)](1-\sigma)\overline{y}a \leq \gamma. \tag{40}$$

Third, suppose that the type of loan contract with $\alpha = 1$ is the IS-1. Then, the maximized value is

$$\begin{aligned} V_{\alpha=1} &= m(\overline{y}a - \gamma) \\ &< m\overline{y}a - m\eta(1-\sigma)\sigma\overline{y}a \\ &\leq [m - \eta(1-\sigma)(m-1)]\overline{y}a \\ &\leq [m - (m-1)(1-\alpha)(1-\sigma)\eta]\,\overline{y}a = V_{IIS\text{-}1}, \end{aligned}$$

where we use the condition of (37) to obtain the first inequality, and $\sigma \geq \frac{m-1}{m}$ to obtain the second inequality. Thus, if the type of loan contract with $\alpha = 1$ is the IS-1, then the IIS-1 type loan contract dominates tree sales.

Combining (37) - (40), we obtain the necessary condition for tree sales to be optimal as $Max\left\{\frac{(m-1)(1-\sigma)\overline{y}a}{m-1+m(1-\sigma)}, [1-\eta(1-\sigma)](1-\sigma)\overline{y}a\right\} \leq \gamma$. Given this condition, tree sales are optimal for all $\alpha \leq \frac{\gamma-[1-\eta(1-\sigma)](1-\sigma)\overline{y}a}{\eta(1-\sigma)^2\overline{y}a}$, which finishes the proof of 2-b). ■